

The sound of a harp string

Nicolas Lynch-Aird & Jim Woodhouse

Version 2a, 22/06/2024

This article explores how the mechanical properties of the different materials used for non-metallic harp strings affect the way they sound, and offers at least a partial explanation of why gut strings sound so much better than nylon, and why fluorocarbon strings provide a much better alternative to nylon, but still don't sound the same as natural gut strings.

The fundamental frequency or pitch of a musical instrument string depends on the string length, diameter, density, and tension [1].¹ For any given string on the harp, the string length and the required frequency are fixed, while the string density, diameter, and tension are linked; so that choosing any two determines the third.

The string density is determined by the choice of string material: nylon strings have a density about 15% lower than natural gut strings, while fluorocarbon strings (often colloquially referred to as 'carbon strings') have a density about 35% higher than gut strings.² The choice of string material, and hence density, immediately fixes the *tensile stress*, which is the ratio of the string tension to its cross-sectional area. Tensile stress is the quantity that determines when a string will break: each material has a limit, the breaking stress. Figure 1 plots the tensile stress, measured in MPa,³ against the string number⁴ for gut, nylon, and fluorocarbon strings. The plot has been arranged so that the thickest strings are on the left. It can immediately be seen how the use of a higher density string material, such as fluorocarbon, increases the tensile stress of the strings. The graph also shows how the stress increases along the string scale, with the highest strings operating under the greatest stress.

The string lengths used for this study were taken from a Russian-made Elysian Cecilia 46 string pedal harp. The string lengths on a more modern concert harp are slightly longer, so that the strings are operating at even higher stress levels. The top gut strings operate very close to their breaking stress limit; this is one of the reasons they break so readily. The breaking stress limit for nylon strings is fairly similar to that for gut, but the lower density of nylon means that nylon strings end up operating comfortably below their breaking point. This explains why many concert harps are strung with nylon for the top octave: they last longer because they are not operating so close to their breaking points.

Estimating the breaking stress for the different string materials is, however, complicated by the fact that the breaking stress varies depending on how quickly the string is being stretched [3].

¹ Numbers in square brackets indicate the references listed at the end of the document.

² The Bowbrand gut strings studied were made from cow gut and had a fairly consistent density of around 1320 kg/m³ [2, 3]. The nylon strings studied fell into two groups: the thinner strings had a density of around 1080 kg/m³, while the thicker strings had a slightly higher density of around 1150 kg/m³ [4]. The fluorocarbon strings had a density of around 1800 kg/m³ [2].

³ The Pascal (Pa) is the unit used to measure stress and pressure and is the ratio of tension, measured in Newtons (N), to area, measured in square-metres (m²). One MegaPascal (MPa) is a million Pascals.

⁴ Concert harps have seven strings per octave. The two highest strings are typically G7 (3136 Hz) and F7 (2794 Hz) on the piano scale, and these are numbered 00 and 0 respectively. The next seven strings, starting from E7 (2637 Hz), are referred to as the 1st octave, and are numbered 1 to 7. The octave and string numbering then continue steadily down the string scale. This article is only concerned with the non-metallic strings, the lowest of which is around the 5th octave A, string 33 (A2, 110 Hz).

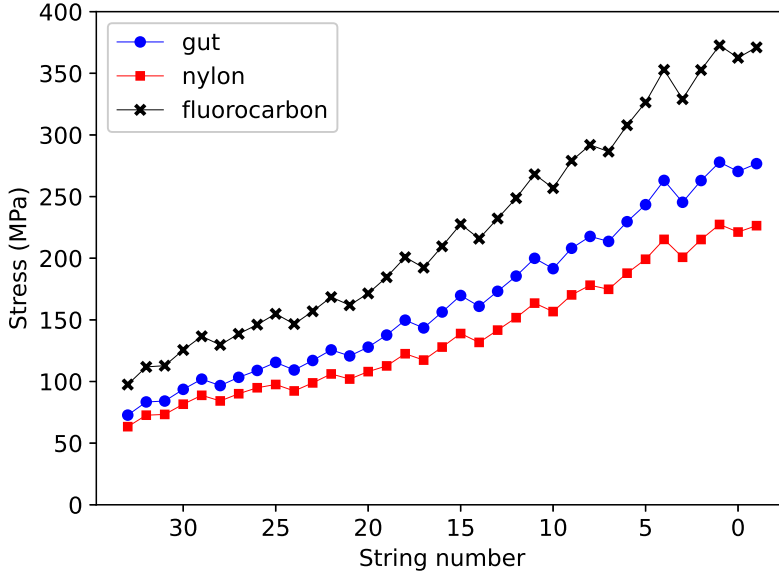


Figure 1: The tensile stress plotted against the string number for gut, nylon, and fluorocarbon strings. The plot has been arranged so that the thickest strings are on the left.

When the strings are stretched they undergo *creep*,⁵ making them weaker. The slower the rate at which the string is stretched, the more time there is for this creep to occur, which lowers the breaking stress. For nylon strings this does not appear to be a significant problem; even after they have been on the harp for some time the top nylon strings should still be able to withstand the stresses they are under. For gut strings, and possibly even more so for fluorocarbon strings, the breaking stress limits can fall far enough, as the strings creep, that eventually the strings can no longer withstand the stresses they are under, and they break. For gut strings this weakening process can be further accelerated by episodes of higher humidity, which can trigger additional episodes of string creep [3, 5]. It is important to note that the tensile stress is entirely independent of the thickness of the string; it depends only on the string length and frequency, and the density of the string material. Consequently, using thicker or thinner strings will not make them less prone to breaking.

For any given string and choice of material, the resulting stress value fixes the ratio of the string tension and cross-sectional area; increasing the thickness of the string will result in a higher tension such that the string may sound louder but will be harder to pluck, while reducing the string diameter too far will result in the string being too ‘floppy’. All of this means that there is only a limited range of string diameters that can be used satisfactorily for any given string position and material: a nylon string will generally be a bit thicker than the equivalent gut string, while a fluorocarbon string will be thinner.

A quantity often used to put a number on this is the ‘feel’, defined as the force required to produce a given displacement at the mid-point of the string. In other words, ‘feel’ describes how hard it is to pull the string.⁶ The need to provide a fairly consistent feel along the string scale, together with a pull range consistent with the string spacing, further constrains the string tension such that in practice it increases almost proportionally with the length of the string.

⁵ Natural and synthetic polymers, such as gut, nylon and fluorocarbon, are what are termed viscoelastic materials. They exhibit both instantaneous, or near-instantaneous, elastic stretching and slower ‘viscous’ stretching. This slower stretching component, which can take many days to settle, is commonly referred to as “string creep”, and is the reason why a new string has to be retuned repeatedly before it stabilises.

⁶ Players do not usually pluck the string precisely at its mid-point, but this measure still provides a useful way to compare neighbouring strings, while being straightforward to calculate.

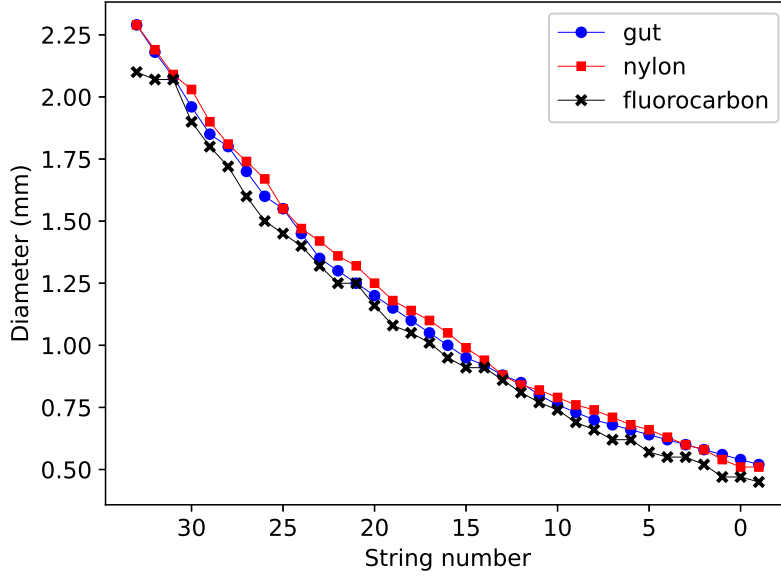


Figure 2: The string diameters used for this study plotted against the string number.

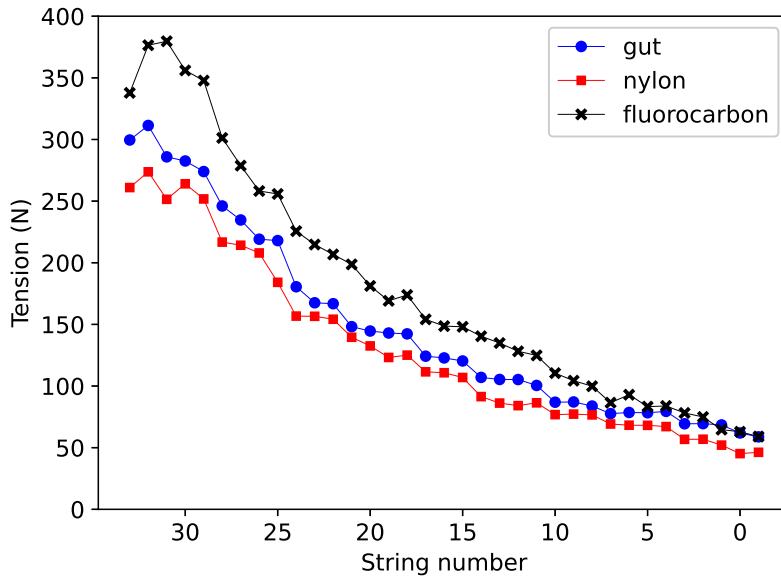


Figure 3: The expected string tensions plotted against the string number.

For this study the various calculations and plots have used the string gauges specified for Bowbrand's Pedal Standard gut and nylon strings,⁷ and Savarez's Pedal Standard range for fluorocarbon strings.⁸ Figures 2 and 3 show the chosen string diameters and the resulting tension values, plotted against the string number for the three materials. As expected, the nylon strings are slightly thicker than the gut strings, while the fluorocarbon strings are a bit thinner. The tensions show the opposite relationship, being higher for fluorocarbon and lower for nylon. Both the string diameter and tension fall along the string scale, although the combination of the two must guarantee that the tensile stress has the opposite trend, as shown

⁷ Bowbrand, August 2023

⁸ Savarez website, August 2023

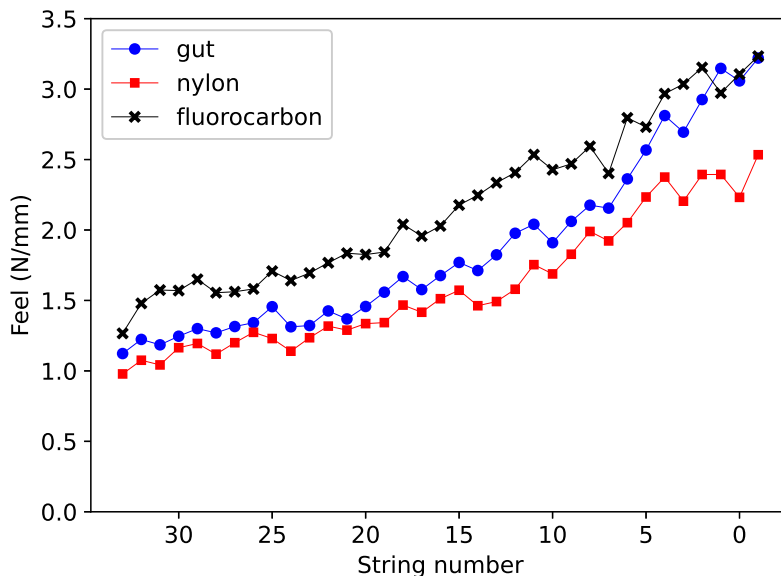


Figure 4: The string ‘feel’ plotted against the string number.

in Figure 1.

Figure 4 shows the string feel, measured in Newtons of applied force⁹ per millimetre of achieved displacement, again plotted against the string number. As players surely require, the feel varies fairly smoothly and over a relatively small range. The higher tensions used for the fluorocarbon strings, however, mean that they will still have a higher feel and will be harder to pluck, while the nylon strings will be easier to pluck. It is interesting to note that the feel of the gut and nylon strings is fairly similar around the bottom end of the 1st octave (string number 7), suggesting that nylon can be used for the top strings without imposing any particularly noticeable transition, in terms of feel, between the nylon and gut sections.

The sound a string makes consists, of course, of much more than its fundamental. For an ideal fully flexible string mounted on a rigid frame, the overtones would be exact integer multiples, the harmonics, of the fundamental frequency. Real strings, however, are not fully flexible; they have a bending stiffness that affects the overtones by making them progressively sharper as the harmonic number increases [6].¹⁰ The overtone frequencies are also slightly perturbed by the fact that the soundboard of the harp is not rigid: of course, if the soundboard did not vibrate the harp would be virtually silent.

The string bending stiffness increases with the string diameter. It also increases with a quantity called the Young’s modulus, which is a measure of the stiffness of the string to stretching (and therefore also of the sensitivity to turning the tuning pin). Figure 5 shows the estimated Young’s modulus, measured in GPa,¹¹ for the different string materials; while Figure 6 gives a measure of the resulting contribution to the overtone inharmonicity, the discrepancy between the string overtones and the true harmonics of the fundamental frequency, relative to that for a 3rd octave A gut string.

The Young’s modulus for nylon and fluorocarbon strings increases almost linearly with the stress [2]. Figure 1 shows that the stress increases along the string scale, so the Young’s modulus, and hence the string bending stiffness and overtone inharmonicity, also increase along

⁹ A force of 10 Newtons is roughly the same as the weight of a 1 kg mass.

¹⁰ A demonstration of the effects of increasing the string bending stiffness is available on the Euphonics website: Sound 1 in Section 7.2, <https://euphonics.org/7-2-choosing-strings/>.

¹¹ One GigaPascal (GPa) is a billion Pascals (see footnote 3).

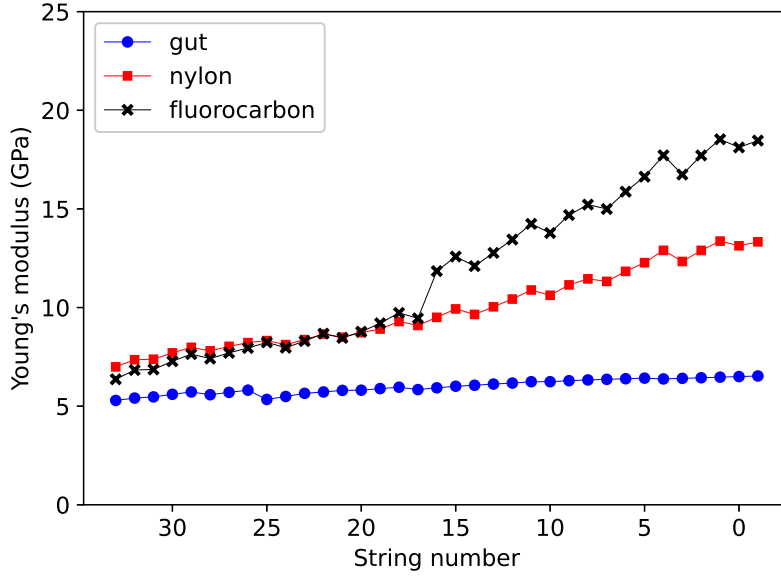


Figure 5: The Young’s modulus of the strings plotted against the string number. The step in the plot for the fluorocarbon strings marks the transition between the overwound and monofilament strings.

the string scale for nylon and fluorocarbon strings. Both fluorocarbon string plots show a step, which marks an important transition: for the thinner strings down to the 3rd octave D (string number 16) the fluorocarbon strings are monofilament strings, formed as a single strand, while for the thicker strings, from the 3rd octave C (string number 17) down, the strings have a monofilament core overwound with a number of thinner strands. This overwound construction significantly increases the string’s flexibility, reducing the Young’s modulus and inharmonicity while maintaining the required overall thickness, and is an approach used in different forms across a wide variety of string types. The nylon strings for this study were all monofilaments.

In contrast with the synthetic materials, the Young’s modulus for natural gut strings stays pretty much constant as the stress is increased, right up to the point where the string breaks [2]. The Young’s modulus for gut strings does vary, however, with the degree of twisting: increasing the degree of twisting reduces the string’s Young’s modulus making it more flexible, but also weakens it [3]. Gut string makers therefore have to make a compromise between reducing the string bending stiffness, and the resulting inharmonicity of its overtones, and maintaining sufficient strength. Consequently, the thicker strings, which are operating well below their breaking stress limit, are twisted to a higher degree than the thinner strings, which need to be kept as strong as possible. The end result is that the Young’s modulus and overtone inharmonicity for gut strings again increase along the string scale.

Figure 5 shows that the Young’s modulus of the gut strings is nevertheless significantly lower than that for nylon and fluorocarbon strings across the whole string scale. The Young’s modulus values are fairly similar for the nylon and overwound fluorocarbon strings, but significantly higher for the thinner monofilament fluorocarbon strings. The relative inharmonicity plots in Figure 6 show that the nylon strings are now significantly worse than both the gut and fluorocarbon strings across the whole string scale. The difference here is primarily due to the thicker diameter of the nylon strings, compared to the other materials. The gut and (thicker) overwound fluorocarbon strings show very similar levels of inharmonicity; the higher Young’s modulus of the fluorocarbon strings, relative to gut, has been offset by their thinner diameters. This changes for the (thinner) monofilament fluorocarbon strings, where the gut strings show the lowest levels of inharmonicity.

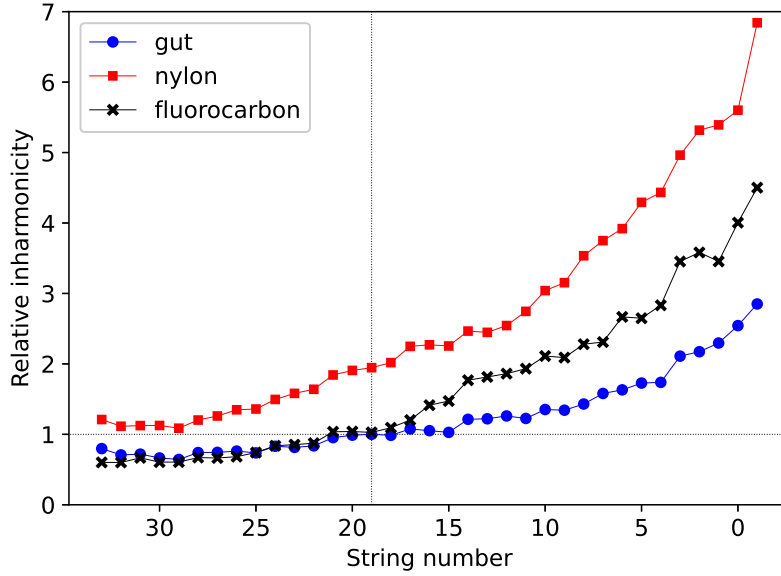


Figure 6: The extent of overtone inharmonicity, relative to that for a 3rd octave A gut string, plotted against the string number.

The string bending stiffness also affects the rate at which the overtones decay, and is the dominant cause of damping at higher frequencies [7]. Figure 7 shows the estimated number of overtones exceeding an appropriate ‘ringiness’ threshold, in other words the number having sufficiently low damping to sound ‘musical’.¹² Figure 8 shows the corresponding effective band-

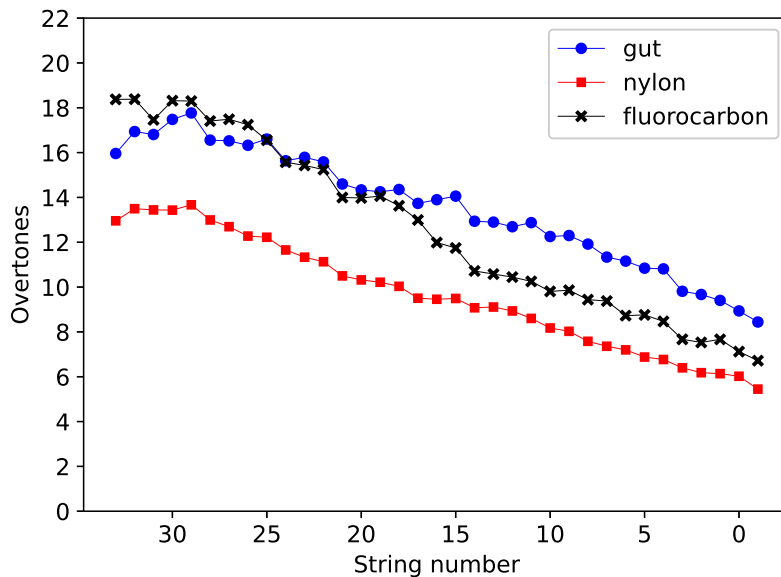


Figure 7: The number of overtones exceeding an appropriate damping threshold plotted against the string number.

¹² The derivation of this damping criteria is given in reference [7], and also in Section 7.2.2 of the Euphonics website, <https://euphonics.org/7-2-2-the-damping-criterion-for-string-selection/>, in the context of constructing a series of string selection charts to assist in selecting the string materials and diameters for a range of plucked string instruments.

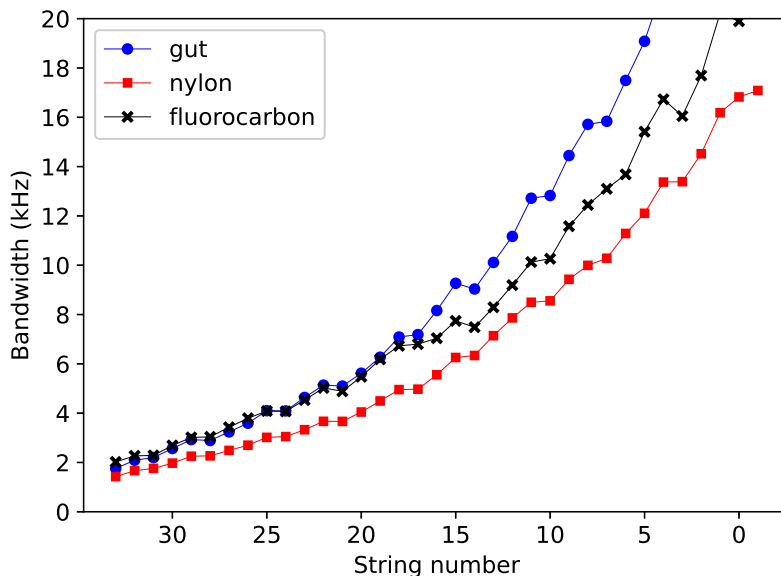


Figure 8: The effective bandwidth of the string plucks plotted against the string number.

width of the sound from plucking the string: higher bandwidth means brighter sound. Again, the results for the overwound fluorocarbon strings are similar to those for gut strings, but the monofilament fluorocarbon strings will have fewer significant overtones and correspondingly lower bandwidths.

These figures, in conjunction with Figure 6, show quite clearly why nylon strings are likely to be considered inferior: compared to both fluorocarbon and gut strings they will generate fewer significant overtones, and also higher levels of inharmonicity. They can be expected to sound relatively dull, and with a more imprecise sense of pitch. Even in the top octave, where the limited bandwidth of nylon strings will be less important due to the limits of human hearing, nylon strings may still not sound as ‘clean’ as the equivalent gut strings; if the latter can survive the tensile stresses they are under.

These results further suggest that the monofilament fluorocarbon strings are also likely to sound inferior to gut, but to a lesser extent. For the overwound fluorocarbon strings, however, the results appear to show that fluorocarbon could provide a worthy alternative to gut. Indeed, it seems likely that the diameters of the fluorocarbon strings have been chosen specifically to give a sound as close as possible to that of the equivalent gut strings. From Figure 4, though, the transition between an upper gut string section and a lower fluorocarbon string section might feel rather noticeable.

For the harp, however, there is a third effect to take into account, again related to the string stiffness. The effects discussed so far all relate to what are termed the transverse modes of the string vibration, with the string vibrating in a direction perpendicular to its length. This is the motion that the player directly excites by plucking the string. But strings can also vibrate *longitudinally*, with the string stretching and compressing along its length rather than moving from side to side.

For instruments such as the guitar and violin families, where the strings run more or less parallel to the soundboard, the longitudinal modes of vibration can be largely ignored, although in the case of the piano they can be important for the sound.¹³ However, two features of the harp make their effects unusually significant. Firstly, the angle between the strings and the

¹³ A rather dramatic demonstration of the effects of changing the tuning of the longitudinal modes of a piano string is available here: https://www.speech.kth.se/music/5_lectures/conklin/longitudinal.html

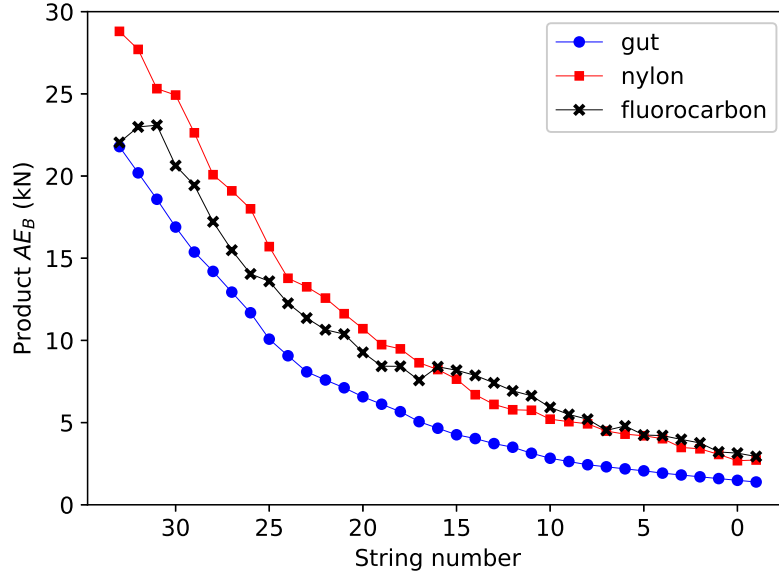


Figure 9: The product of the string cross-sectional area and its Young’s modulus, which provides a measure of the relative amplitude of the phantom partials, plotted against the string number.

soundboard means that any longitudinal vibration in the strings will drive vibration of the soundboard much more effectively than in, say, a guitar. Soundboard vibration in turn creates sound radiation in the surrounding air, so the geometry of the harp makes longitudinal string vibration more audible.

Secondly, longitudinal string vibration is primarily caused by a mechanism that is encouraged by another important feature of harp playing: the relatively large displacements of harp strings, compared to many other plucked string instruments. This leads to a phenomenon referred to as “phantom partials” [8, 9]. These are additional frequency components which occur most strongly at frequencies which are either double those of the transverse overtones, or the sum of adjacent transverse overtones.¹⁴ The amplitude of these phantom partials varies with the *square* of the transverse displacement of the string, so the large displacement of a strongly-plucked harp string means that the phantom partials can be strongly excited.

The amplitude of these phantom partials also depends on the string tension and on the product of the string cross-sectional area and its Young’s modulus. This latter quantity, measured in kN,¹⁵ is shown in Figure 9. Looking back to Figure 3, both the string tension and this new quantity are higher for fluorocarbon strings than for gut strings across the whole of the string scale.

Since the transverse overtones are subject to the inharmonicity due to the string bending stiffness, they will not be true harmonics of the string fundamental frequency. Worse, since the degree of the inharmonicity increases with the harmonic number, the frequencies of the phantom partials will be somewhat lower than those of the equivalent transverse overtones.¹⁶ This means that the noticeability of these phantom partials will depend on both their amplitude and the degree of inharmonicity of the transverse overtones, both of which will be larger for the

¹⁴ A demonstration of the effects of phantom partials on the sound of the harp is available on the Euphonics website: Sounds 2 to 7 in Section 7.4, <https://euphonics.org/7-4-add-a-touch-of-nonlinearity/>.

¹⁵ The product of the cross-sectional area and Young’s modulus has units of force, measured in Newtons (see footnotes 3 and 11). One kiloNewton (kN) is a thousand Newtons.

¹⁶ Section 7.4 of the Euphonics website, especially Figure 5, demonstrates this in some detail, <https://euphonics.org/7-4-add-a-touch-of-nonlinearity/>.

monofilament fluorocarbon strings compared to natural gut strings. For overwound fluorocarbon strings, while the degree of inharmonicity may be similar to that for gut strings, the amplitude of the phantom partials will be higher, potentially still making them more noticeable for the fluorocarbon strings.

The relative contribution from the phantom partials could be reduced for fluorocarbon strings by reducing the string diameters and tensions, but not without changing the overall brightness of the strings.¹⁷ Similar changes could also be made for gut strings by reducing the string diameters, but gut string makers also have the option of adjusting the string sound by varying the degree of twist during manufacturing, while leaving the string diameters, tensions and feel unchanged.

As a final note, it should be pointed out that this study has not taken into account the effects of the variations that inevitably occur along the length of a natural gut harp string. Overall, however, it is clear that fluorocarbon strings can get much closer to replicating the sound of gut strings than nylon strings can, but fluorocarbon and gut strings will still not sound the same. Ultimately, the choice of which strings to use will be down to individual preference.

References

- [1] D. J. Benson. *Music: A Mathematical Offering*. Cambridge University Press, Cambridge, 2007. p. 166.
- [2] N. J. Lynch-Aird and J. Woodhouse. Comparison of mechanical properties of natural gut and synthetic polymer harp strings. *Materials*, 11(11):2160, 2018. doi:10.3390/ma11112160.
- [3] N. J. Lynch-Aird and J. Woodhouse. Effects of twisting and surface finish on the mechanical properties of natural gut harp strings. *Materials*, 16(15):5444, 2023. doi:10.3390/ma16155444.
- [4] N. J. Lynch-Aird and J. Woodhouse. Mechanical properties of nylon harp strings. *Materials*, 10(5):497, 2017. doi:10.3390/ma10050497.
- [5] N. J. Lynch-Aird and J. Woodhouse. The effects of humidity changes on natural gut harp strings. *Proceedings of Meetings on Acoustics*, 43(1):035002, 2021. URL: <https://asa.scitation.org/doi/abs/10.1121/2.0001441>, doi:10.1121/2.0001441.
- [6] N. H. Fletcher and T. D. Rossing. *The Physics of Musical Instruments*. Springer, New York, 2nd edition, 1998.
- [7] J. Woodhouse and N. J. Lynch-Aird. Choosing strings for plucked musical instruments. *Acta Acustica united with Acustica*, 105(3):516–529, 2019. URL: <https://www.repository.cam.ac.uk/handle/1810/294170>, doi:10.3813/AAA.919333.
- [8] J. Woodhouse. The acoustics of a plucked harp string. *Journal of Sound and Vibration*, 523:116669, 2022. URL: <https://www.sciencedirect.com/science/article/pii/S0022460X21006817>, doi:10.1016/j.jsv.2021.116669.
- [9] H. A. Conklin. Generation of partials due to nonlinear mixing in a stringed instrument. *Journal of the Acoustical Society of America*, 105:536–545, 1999. doi:10.1121/1.424589.

¹⁷ Sound 2 in Section 7.2 of the Euphonics website provides a demonstration of the effect of changing the string diameter, <https://euphonics.org/7-2-choosing-strings/>.

Version history

Version	Date	Comments
1	15 Dec 2023	First release
2	15 May 2024	Updated Figures 5 to 9 using improved estimates for Young's modulus of gut strings
2a	22 June 2024	Clarifications following expert reader feedback
