

Studies of the Sensitivity of Brake Squeal

T. Butlin^{1,a} and J. Woodhouse^{1,b}

¹Cambridge University Engineering Department,
Trumpington Street, Cambridge, CB2 1PZ, England

^atb267@cam.ac.uk, ^bjw12@cam.ac.uk

Keywords: Brake noise, friction induced vibration, sensitivity, uncertainty, pin-on-disc.

Abstract. The problem of ‘brake squeal’ in the automotive industry remains despite over 70 years of research: the phenomenon is still surprisingly unpredictable and poorly understood. The literature has moved from very simple lumped parameter models to ever more sophisticated finite element models, but testing theory against measurements has been hindered by the difficulty in obtaining repeatable results. It would seem the phenomenon is extremely sensitive to changes in parameters beyond an experimenter’s control.

This paper describes recent results from a project to identify and quantify the sources of uncertainty within sliding contact systems and to determine the sensitivity of the friction-coupled system to uncertain parameters. The theoretical approach taken is to use a linear analysis based on the uncoupled transfer functions of two general subsystems to predict stability when they are coupled by a sliding point contact. The model is tested using a pin-on-disc rig whose uncoupled transfer functions can be measured.

Using a stability criterion based on the roots of the characteristic equation of the system, the sensitivity of model predictions to parameter variations is investigated numerically. It is shown that using a realistic range of parameters the root locations change considerably and enough to change stability predictions. As the complexity of the model is increased reliable results become harder to achieve as the characteristic equation becomes more ill-conditioned. This is not simply a result of the high order of the system, but is thought to be a result of particular mode combinations. Experimental work shows uncoupled transfer functions vary over time and by enough to significantly affect squeal predictions. These results suggest reasons for the difficulty in obtaining repeatable measurements and for the unreliability of squeal prediction theories developed so far. If the reasons for the sensitivity of squeal can be understood it may be possible to design sliding contact systems that are more robust.

Introduction

Friction-induced vibration is a phenomenon that arises over a diverse range of scales and contexts, and includes the noise that sometimes occurs when a vehicle is braking (mostly referred to as ‘brake squeal’). Initial research focussed on using the frictional properties of the contacting materials to predict the occurrence of squeal. While it is clearly important to understand the tribology of the system it is equally important not to neglect the surrounding system properties. Spurr [1] was the first to propose a mechanism for how the supporting structure of a sliding contact system could play a part in the generation of ‘stick-slip’ oscillation, which led to increasingly complex lumped parameter models. Linearising the system produces an eigenvalue problem for the complex ‘natural frequencies’: a particular vibration mode is unstable if the imaginary part of the eigenvalue is negative. The growth in amplitude will be limited in practice by some non-linearity not accounted for in the model such as the onset of stick-slip motion. Squeal research has been summarised by North [2] up until 1976, Ibrahim [3] more recently and Kinkaid [4] for the last decade. North [2] noted that estimating equivalent modal masses and stiffnesses proved extremely difficult, but finite element and multibody dynamics analysis packages have made it possible to model complete brake systems giving the potential to relate the models to real systems more easily than with lumped parameter models — Kinkaid *et al.* [4] provide a useful review of developments up until 2003. However, a recurring theme of research to date is the difficulty in obtaining repeatable results that correlate with theoretical models — even the most sophisticated models have only suggested ‘fixes’ for squeal, not general design principles and there is still no validated predictive model of friction-induced vibration. The

closest that has been achieved so far is in the context of the behaviour of the bowed violin string (e.g. the review by Woodhouse and Galluzzo [5]).

The present research is based on the formulation of Duffour [6]. His approach was to combine modal analysis with linear stability theory to develop a method for predicting the stability of two systems coupled by a single point sliding contact. Formulating the point contact problem in terms of the transfer functions of each of the uncoupled subsystems solved (in part) the problem of relating parameters to a real system, since transfer functions are measurable. The theory was tested on a pin-on-disc apparatus. The results showed a reasonable correlation between squeal frequencies and predicted unstable modes, however they were not completely conclusive.

The universal observation that repeatable results are difficult to obtain suggests that friction-coupled systems are highly sensitive to parameter changes. It is the aim of this research to identify the cause of this sensitivity and to determine whether a sliding contact system can be designed to be more robust. This paper describes recent results from numerical sensitivity studies and from experimental work on the pin-on-disc rig carried out to test this hypothesis.

Summary of Theoretical Framework

For a complete description of the theoretical framework and experimental test rig, see Duffour [6]. The system to be analysed is sketched in Fig. 1. The 'disc' is driven at constant velocity, V_0 , and the 'brake' is pushed against it with a dynamically varying normal force, N , composed of a steady equilibrium preload, N_0 , plus a small fluctuating component, N' , such that $N = N_0 + N'$. The normal and tangential displacements from equilibrium of the disc are denoted u_1 and v_1 respectively, and u_2 and v_2 for the brake. The normal and tangential displacements from equilibrium of the point of contact are denoted u_3 and v_3 . The spring of stiffness k_n represents the linearised contact stiffness. Any damping that may result from the contact has initially been ignored.

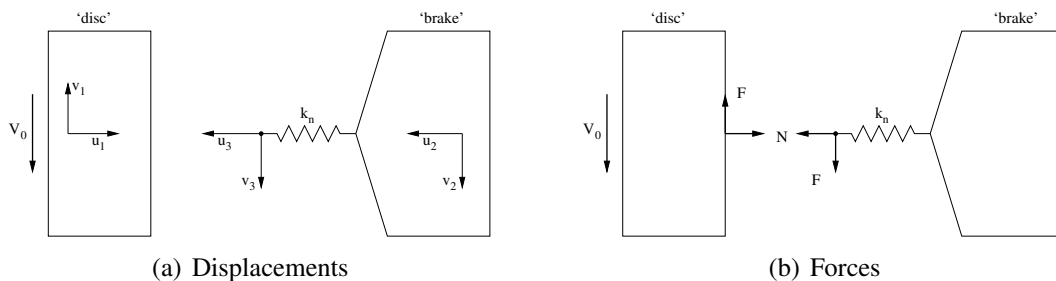


Figure 1: Two linear subsystems coupled by a single point sliding contact, with definition of variables.

The dynamics of the 'disc' and 'brake' can be described in terms of transfer functions:

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} G_{11}(\omega) & G_{12}(\omega) \\ G_{21}(\omega) & G_{22}(\omega) \end{bmatrix} \begin{bmatrix} N' \\ F' \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix} \begin{bmatrix} N' \\ F' \end{bmatrix}, \quad (2)$$

where $G_{ij}(\omega)$ are the transfer functions representing the disc's response and $H_{ij}(\omega)$ represent the equivalent set of responses for the brake. These transfer functions can be determined using standard vibration measurement techniques.

Assuming a constant coefficient of friction the characteristic equation of the sliding coupled system is given by Eq. 3. This system will be unstable if and only if the function $D(\omega)$ has at least one zero with a negative imaginary part, where

$$D = G_{11} + \mu G_{12} + H_{11} + \mu H_{12} + 1/k_n. \quad (3)$$

If a coefficient of friction that varies with sliding velocity is now included, the relationship between F and N can be linearised such that $F \approx [\mu_0 + i\omega\epsilon(v_1 + v_2)]N$. The factor $i\omega$ converts the displacements v_1 and v_2 into velocities and ϵ is the linearised gradient of the $\mu - v$ curve. More generally, if ϵ is allowed to become complex and frequency dependent then it could describe any linearised relationship between F and N . The characteristic equation is now given by Eq. 4 (see Duffour [7]):

$$E = D - i\omega\epsilon \left[(G_{11} + H_{11})(G_{22} + H_{22}) - (G_{12} + H_{12})^2 \right]. \quad (4)$$

Numerical work

Some numerical tests were carried out to explore the sensitivity of predictions to changes in the model parameters. Initially, the coefficient of friction was assumed to be constant. A Matlab function was written to calculate the characteristic equation of the system and its roots given the coefficient of friction, μ , the velocity dependence, ϵ , the contact stiffness, k_n , the rotation of the pin relative to the disc, θ (to take into account misalignments when bringing the pin and disc into contact) and the modal parameters from the measured transfer functions: the natural frequencies, ω_n , amplitudes, a_n , and damping factors, Q_n . If the system has n modes then the characteristic polynomial is of order $2n$. When a varying coefficient of friction is included the order rises to $4n$.

Numerical studies have been made in which the system parameters were varied over a realistic range. Fig. 2(a) shows a cloud plot for the case where $0.4 < \mu < 0.6$, $\epsilon = 0$, $-1^\circ < \theta < 1^\circ$, $0.9 \times 10^6 < k_n < 3.6 \times 10^6 \text{ Nm}^{-1}$, and the modal parameters of each pin mode were varied by 10% from their measured values. Fig. 2(b) shows the root variation when $0 < \epsilon < 0.01$.

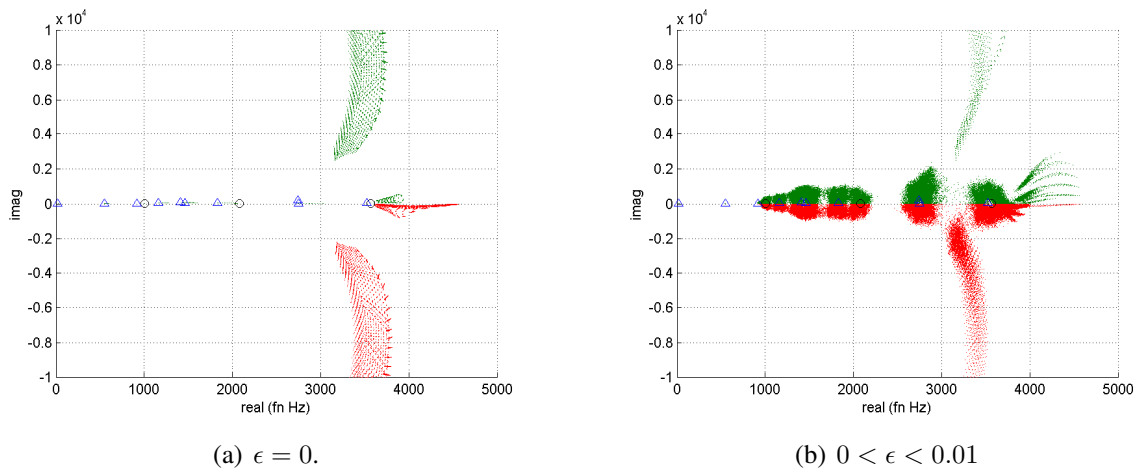


Figure 2: Cloud plot of roots as parameters vary. ‘O’ and ‘ Δ ’ represent the uncoupled poles of the disc and pin respectively.

It is interesting to note that the roots change enormously — if the measurements are as uncertain as the range covered here then it is no surprise that squeal predictions are very unreliable. The broad region in the middle of Fig. 2(a) shows that the imaginary part of the zeros can move by up to 600% while the real part remains relatively unchanged. The root near 4 kHz can be either stable or unstable and the predicted squeal frequency if unstable varies by up to 25%. It is also interesting to observe the variation in density of root locations — this implies a sensitivity that varies with changes in parameters. Adding a velocity dependent coefficient

of friction increases the effect of uncertainties such that predictions made on the basis of a measurement are highly unreliable.

To explore the parameter sensitivity further, a Matlab function was written to find the sensitivity as any two parameters were varied. The algorithm samples the variable space, (v_1, v_2) , by calculating the phase of the root locations at three close intervals for each variable sample, and obtaining an approximation to the gradient at each point according to Eq. 5:

$$|\nabla\phi| \approx \sqrt{\frac{\partial\phi^2}{\partial v_1} + \frac{\partial\phi^2}{\partial v_2}}, \quad (5)$$

where

$$\frac{\partial\phi}{\partial v_1} \approx \frac{(\angle z_2 - \angle z_1)}{\delta v_1}, \quad \frac{\partial\phi}{\partial v_2} \approx \frac{(\angle z_3 - \angle z_1)}{\delta v_2}. \quad (6)$$

Fig. 3(a) shows how the sensitivity of one root changes as the natural frequency of the first disc mode and third pin mode are varied by $\pm 50\%$. These modes were chosen as they were already fairly close together with natural frequencies of $f_{disc1} = 1027$ Hz and $f_{pin3} = 916$ Hz. A maximum occurs when $f_{disc1} \approx f_{pin3} \approx 1200$ Hz. The maximum at $f_{pin3} = 1374$ Hz also lies close to the line defining $f_{disc1} = f_{pin3}$. It would seem therefore that sensitivity is increased when the natural frequencies of uncoupled modes are close together, which might explain some of the observations in the literature that brake squeal is more likely in these situations — perhaps it would be more accurate to say that it is less predictable. However this does not explain why the sensitivity changes along the line for which the natural frequencies are close, nor what causes the other features.

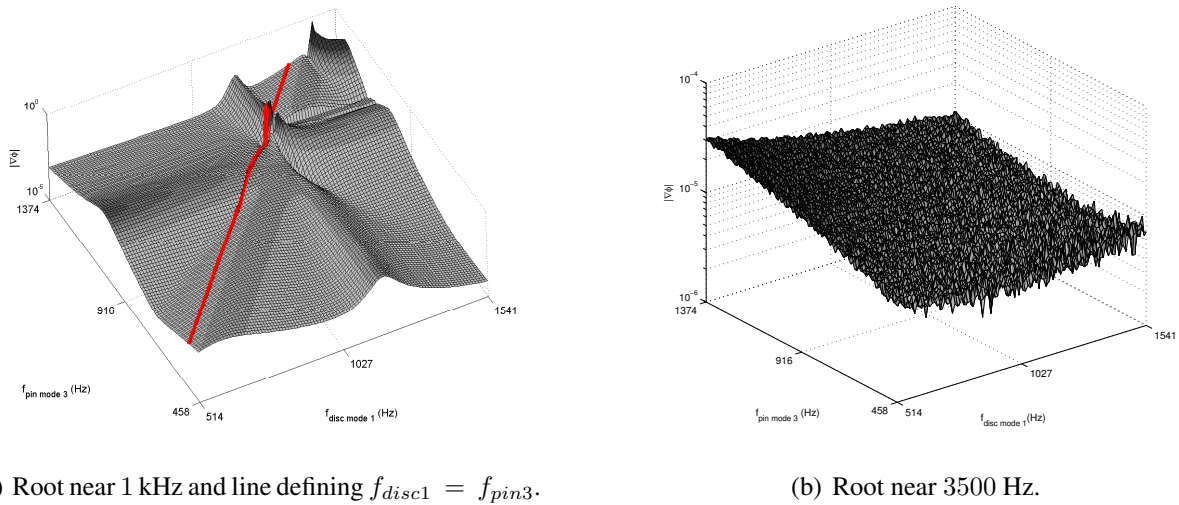


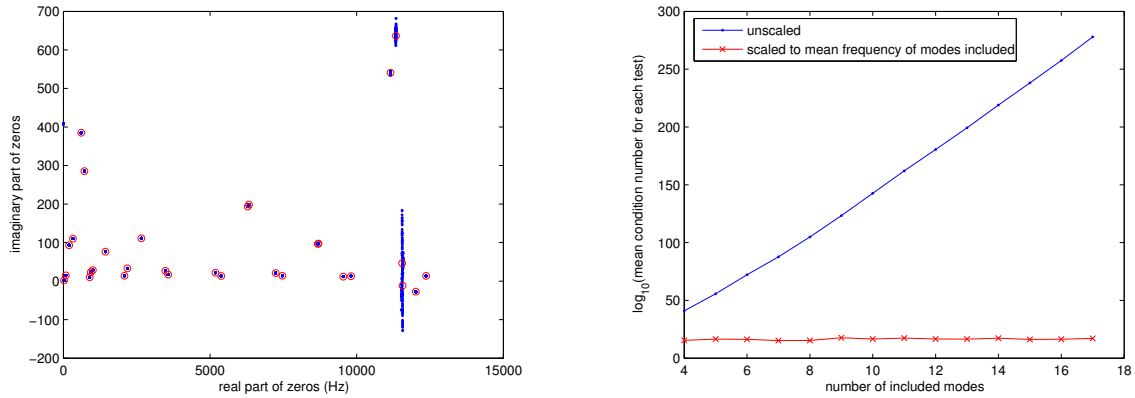
Figure 3: Variation of sensitivity of roots

One difficulty encountered was ‘noisy’ surfaces — see for example Fig. 3(b). It can be seen that the sensitivity data is not giving as smooth results as in Fig. 3(a). It is still useful as overall trends are visible, but the noise level becomes more dominant with increasing model complexity.

Including a coefficient of friction that varies linearly with velocity doubles the order of the characteristic equation and the number of roots, so it is not surprising that it has a significant effect on predictions. Also, the roots become extremely sensitive to perturbations of the polynomial coefficients. Even perturbations of the order of finite double-precision arithmetic of Matlab resulted in very large changes in some root locations (see Fig. 4(a)). When the root-finding code was used in the function to determine sensitivity as two parameters varied, the results showed few trends as they were dominated by finite precision errors, but it is interesting to note that only some of the roots are highly sensitive which again points towards parameter-dependent sensitivity.

A measure of the numerical sensitivity of the system can be found by rewriting the problem in terms of eigenvalues — the roots of a polynomial are also the eigenvalues of its companion matrix, A . The sensitivity of

each eigenvalue, s_i , is given by $s_i = (y_i^T x_i)^{-1}$, where x_i and y_i are the corresponding eigenvectors of A and A^T respectively. For more details see Wilkinson [8]. The dots of Fig. 4(b) show the mean condition number, s_{mean} , as the number of modes included increases. It appears that the system becomes more ill-conditioned. However, the crosses show s_{mean} when each test is scaled by the mean of the natural frequencies included. This makes the problem much better conditioned and almost independent of the number of modes included.



(a) Variation of roots with 100 random small perturbations of the polynomial coefficients. Circles represent roots of the unperturbed polynomial.

(b) Effect of increasing the number of modes upon the mean condition number of each test.

Figure 4: Numerical sensitivity.

In order to explore more systematically the cause of high sensitivity, a three mode case was considered:

$$D = \frac{a_1}{\omega_1^2 + 2i\zeta_1\omega_1\omega - \omega^2} + \frac{a_2}{\omega_2^2 + 2i\zeta_2\omega_2\omega - \omega^2} + \frac{a_3}{\omega_3^2 + 2i\zeta_3\omega_3\omega - \omega^2} \tag{7}$$

Fig. 5 shows the sensitivity of each root over the range $-1 < a_3 < 1$ and $0.8 < \omega_3 < 1.4$ with $a_1 = a_2 = 1$, $\omega_1 = 1$, $\omega_2 = 1.2$, $\zeta_1 = \zeta_2 = 0.01$, $\zeta_3 = 0.03$ and considering only the simplest case where $k_n \rightarrow \infty$ and $\epsilon = 0$. With $a_3 \approx -1$ there are two maxima close to $\omega = 1$ and $\omega = 1.2$, consistent with the previous observation of increased sensitivity when modes are close in natural frequency. As $a_3 \rightarrow 0$ these maxima begin to converge and increase in magnitude. When $a_3 = 0$ these become one, however, tracking the roots becomes difficult as each pair of roots goes through a single point on the complex plane, so the region where $0 \leq a_3 < 1$ is not completely reliable. Also, when $a_3 \approx -1.5$ (not shown) the roots bifurcate and tracking them through this change is difficult. These sudden changes are extremely interesting and suggest that real systems can change behaviour completely when changed by a small amount under certain circumstances. With an improved tracking algorithm, these effects will be able to be analysed more effectively.

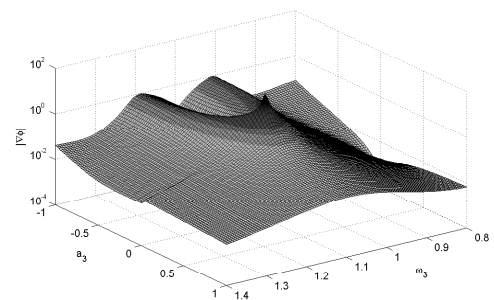
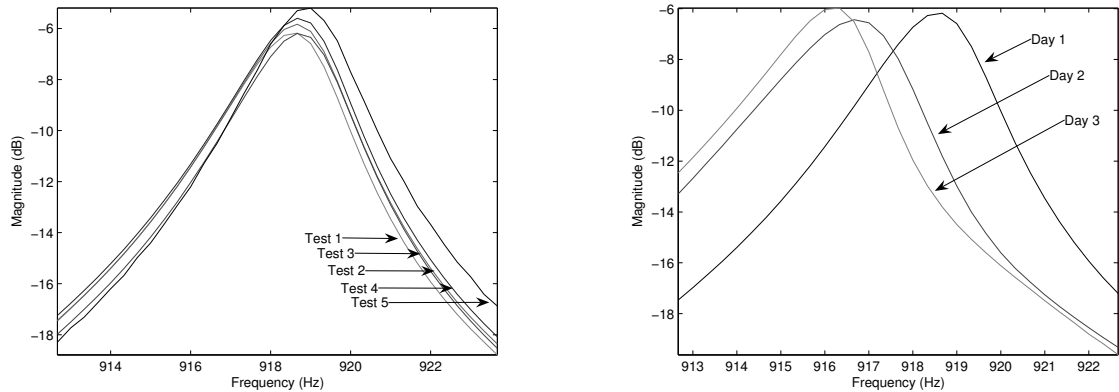


Figure 5: Sensitivity of three mode system to changes in a_3 and ω_3 .

Experimental work

The main aims of the experimental work were to quantify the uncertainties within the system to provide inputs for the numerical studies, and to explore their significance. As seen, the model predicts that the behaviour of the system is highly dependent upon the modal parameters of the uncoupled subsystems and the parameters defining the contact model. These can only be found to a certain precision, and may vary with time. To test the variability of the system dynamics, the transfer functions of the pin and the disc were measured multiple times over several days under nominally identical conditions. The transfer functions were also measured before and after readjusting the clamping bolts, and before and after realignment of the laser vibrometer and the impulse hammer.



(a) Five repeat measurements within 20 minutes.

(b) Three repeat measurements on three different days.

Figure 6: Variation of one peak of $H_{11}(\omega)$.

$H_{11}(\omega)$ was measured five times within 20 minutes. The first two tests were direct repeats without changing anything, the third was after realigning the laser and hammer, the fourth after loosening and tightening the bolts holding the assembly to the table and the fifth on a more sensitive laser vibrometer setting (subsequently compensated for) such that lighter impulses gave the same voltage output. Fig. 6(a) shows how the peak at 919 Hz varies for each of these measurements. Fitting the modal parameters showed that the variation of the peak frequency is only around 0.05%, but the damping factor varies by 2% and the modal amplitude varies by 7%. Various physical effects can account for these deviations. It is possible that realigning the hammer and the laser very slightly varies the modal amplitudes of all of the peaks, so at this peak the residual components of the other modes shifts the resonant frequency very slightly. Changes in the tightness of the bolts could also give unpredictable variations in damping. Also the average impulse amplitude delivered by the hammer is slightly different for each measurement, so any effect of spring hardening would vary each result. Further tests would be required to identify the dominant effect. It is curious however, that even two tests repeated one after the other without any changes made should give even slightly different results. While for most applications such uncertainty is small enough to be neglected, it has been shown in the previous section that these small changes can affect the stability of sliding contact systems.

Fig. 6(b) shows the variation of $H_{11}(\omega)$ measured on three separate days. Now the variations are significantly greater. While the peak frequency shifts by only 0.3%, the spread is almost as large as its bandwidth and thus cannot be accounted for simply on the basis of misalignment. The damping and modal amplitudes are changed by around 20%. The variability could perhaps be the result of changing ambient conditions such as temperature or humidity, though no tests have been carried out here to confirm this. Similar variations were observed for most of the peaks of all the measured transfer functions, though with slightly less variation of the disc transfer functions. Presumably frictional and contact parameters could vary by a larger factor, but this has not yet been explored quantitatively.

Conclusions

Obtaining repeatable results for brake squeal tests is difficult. Experimental work reveals a significant degree of uncertainty and variability of system parameters and numerical studies show that this strongly influences squeal predictions. Initial tests to determine the causes of sensitivity suggest that the coupled system seems to be more sensitive to parameter variation when two uncoupled modes are close in natural frequency. Including a velocity dependent coefficient of friction significantly complicates the model and predictions can be dominated by finite precision errors which are not just a result of the high order of the characteristic equation, but depend on the distribution of the modes. Future work aims to explore these issues in more detail and to isolate the causes of high numerical and parameter sensitivity.

Acknowledgements

Financial support from the EPSRC is gratefully acknowledged.

References

- [1] R. T. Spurr. A theory of brake squeal. *Proceedings of the Institution of Mechanical Engineers (AD)*, 1:33–52, 1961-1962.
- [2] M. R. North. Disc brake squeal. IMechE Conference on Braking of Road Vehicles, 1976.
- [3] R. A. Ibrahim and E. Rivin. Friction-induced vibration, chatter, squeal, and chaos part ii: Dynamics and modeling. *American Society of Mechanical Engineers Applied Mechanics Reviews*, 47(7):227–253, July 1994.
- [4] O. M. O'Reilly N. M. Kinkaid and P. Papadopoulos. Automotive disc brake squeal. *Journal of Sound and Vibration*, 267:105–166, 2003.
- [5] J. Woodhouse and P. M. Galluzzo. The bowed string as we know it today. *Acta Acustica*, 90(4):579–589, Jul-Aug 2004.
- [6] P. Duffour and J. Woodhouse. Instability of systems with a frictional point contact. part 1: basic modelling. *Journal of Sound and Vibration*, 271(1-2):365–390, March 2004.
- [7] P. Duffour and J. Woodhouse. Instability of systems with a frictional point contact. part 2: model extensions. *Journal of Sound and Vibration*, 271(1-2):391–410, March 2004.
- [8] J. H. Wilkinson. *The Algebraic Eigenvalue Problem*. Oxford University Press, 1965.