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The vibration damping of laminated plates

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A method previously developed for determination of elastic and damping parameters of orthotropic plates (McIntyre, M. E. and Woodhouse, J., *Acta Metall.*, 1988, **36**, 1397–1416) was applied to laminated composite plates. The necessary theory is summarised, and the predictions of laminate theory compared with experimental results for three CFRP laminated plates with different constructions. It is also shown that laminate theory can be inverted, to obtain the ply properties from measurements on the laminated plate. This can sometimes afford a good way to obtain the necessary calibration data on the material properties of the plies. © 1997 Published by Elsevier Science Limited.

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INTRODUCTION

The damping of an engineering structure is important in many aspects of noise and vibration control, fatigue endurance and so on, since it controls the amplitude of resonant vibration response. Energy dissipation mechanisms in a structure can be divided into two classes: those associated with the inherent material damping, and those associated with additional sources of dissipation such as friction at joints. In conventional metal structures the latter usually dominates, but with fibre-reinforced plastic structures the number of joints is kept low, and where possible they are bonded. As a result the inherent material damping contributes significantly to the overall damping, and is often the primary means of controlling the structure's dynamic behaviour. It is thus important to be able to control and predict the level of inherent damping in such materials.

Most laminate lay-ups are designed to produce a plate whose overall properties are orthotropic. A method for measuring both the elastic and damping properties of such plates was described in some detail by McIntyre and Woodhouse[1], and illustrated with results from a variety of natural and man-made orthotropic sheet materials. In the present investigation, this methodology is applied to laminated plates. The linear elastic behaviour of such plates can be predicted from the properties of the individual plies using laminate theory. Laminate theory can also be used to predict the damping properties of such plates, through the concept of "complex modulus". This is justified by the "viscoelastic correspondence principle"[2,3], which allows any result of linear elasticity to be extended to a linear viscoelastic material by replacing the elastic moduli with suitable (possibly frequency-dependent) complex quantities.

Central to the measurement method is a result, valid provided the damping is small, which allows the damping of a given vibration mode of a plate (or any other structure) to be calculated from knowledge of the *undamped* mode shape, together with the material damping properties. It was used to predict the effect on modal damping of changes in geometry, boundary conditions, and thickness distribution[4]. The method has the advantage that, once the material damping properties are known in a suitable form, one can calculate modal damping by post-processing of undamped computations, for example based on standard finite-element code.

Other authors have studied damping of laminated plates: for example Lin et al.[5], Hwang and Gibson[6]. The emphasis of this article differs in two ways.

(1) No attempt is made to investigate the micromechanics whereby the overall elasticity and damping of the plies relates to that of the fibres and matrix. Instead, the aim is to illustrate and validate experimentally a simple way in which the effective properties of the plies may be determined, and then used to predict those of any laminate. This is useful for the composite designer, and also the measurement methodology may be useful for non-destructive testing and quality control since manufacturing faults such as incomplete adhesion between plies are likely to show up in modified vibration damping.
(2) The method determines damping properties in a context which allows relatively easily for analytic calculations as well as computation of vibration mode damping of plates with complex shapes and boundary conditions. For example, perturbation methods based on Rayleigh's principle can be applied to determine the sensitivity of

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mode frequencies and damping factors to possible design changes[4]. This is in contrast with methods which are intimately associated with finite-element formulations, which are good for computation but less easily used for sensitivity studies.

OVERVIEW OF THE THEORY

The aim is to predict the damping constants governing a flat, orthotropic laminate given the elastic and damping constants of the constituent plies. The usual assumptions of laminate theory and thin-plate bending theory will be made, and for simplicity the discussion will be phrased in terms of a laminate made from identical plies, with only the orientation varying. The laminates tested experimentally were all of this type.

In the case of a thin, specially orthotropic plate, small-amplitude vibrational behaviour is governed by four elastic constants, for the simplest case in which shear and rotatory inertia are ignored. We choose to use the constants D_1, D_2, D_3 and D_4 introduced by McIntyre and Woodhouse[1,4]. These are defined through the elastic strain energy of the plate as it vibrates in the x-y plane with a centre-plane transverse displacement in the z-direction $w(x,y)e^{i\omega t}$:

$$V=\frac{h^3}{2}\int\int(D_1w_{xx}^2+D_2w_{xx}w_{yy}+D_3w_{yy}^2+D_4w_{xy}^2)\mathrm{d}A \quad (1)$$

where h is the plate thickness, subscripts x and y denote partial derivatives and the integral is taken over the area of the plate. The Ds can be written in terms of the Young's moduli along the two principle directions (x and y axes) of the plate E_x and E_y, the two Poisson's ratios between these directions ν_{xy} and ν_{yx}, and the in-plane shear modulus G_{xy}:

$$D_1=E_x/12\mu \qquad D_2=\nu_{xy}E_y/6\mu=\nu_{yx}E_x/6\mu$$
$$D_3=E_y/12\mu \qquad D_4=G_{xy}/3 \quad (2)$$

where $\mu = 1 - \nu_{xy}\nu_{yx}$. Note that, in broad terms, D_1 is associated with bending in the x direction, D_3 with bending in the y direction, D_4 with twisting motion, and D_2 with Poisson's-ratio coupling between the x and y directions. The influence of this last term is rarely considered in the existing literature of the subject, although under some circumstances it can have a very significant influence.

The essential part of laminate theory can be summarised as follows (see for example Tsai[7]). Consider a general laminate composed of n identical plies of thickness t, and suppose that the ith ply is oriented at θ_i to the laminate axes. The strain energy density in a given ply can be expressed in terms of its strain vector ε^i as

$$\delta V_i=\tfrac{1}{2}(\varepsilon^i)^t\boldsymbol{K}^i\varepsilon^i \text{ where } \boldsymbol{K}^i=\boldsymbol{R}^i\boldsymbol{K}(\boldsymbol{R}^i)^t. \quad (3)$$

Here, $\boldsymbol{K}$ is the ply stiffness matrix (relative to its natural axes) and $\boldsymbol{R}^i$ is a coordinate transformation matrix:

$$\boldsymbol{K}=\begin{bmatrix}12D_1 & 6D_2 & 0\\ 6D_2 & 12D_3 & 0\\ 0 & 0 & 3D_4\end{bmatrix} \quad \boldsymbol{R}^i=\begin{bmatrix}c_i^2 & s_i^2 & -2c_is_i\\ s_i^2 & c_i^2 & 2c_is_i\\ c_is_i & -c_is_i & c_i^2-s_i^2\end{bmatrix} \quad (4)$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$. Assuming linear strain variation through the thickness of the plate, integrating over the thickness of each of the n plies, summing the results and then integrating over the area of the laminate we obtain an expression for the total strain energy of the deformed laminate. In the case of a specially orthotropic laminate, this can be compared directly with eqn (1) to reveal the elastic constants of the laminate in terms of those of the plies:

$$D_j^{\mathrm{lam}}=\frac{4}{n^3}\sum_{i=1}^{n}\alpha_iD_j^i \; j=1,2,3,4 \quad (5)$$

where

$$\alpha_i=\left[\left(i-\frac{n}{2}\right)^3-\left(i-1-\frac{n}{2}\right)^3\right]$$

and the terms D_j^i are defined from certain elements of $\boldsymbol{K}^i$ in the analogous way to eqn (4) (but noting that $\boldsymbol{K}^i$ will not in general have zeros where $\boldsymbol{K}$ has them).

Note that the lay-up needs sufficient symmetry in order that the overall stiffness matrix has zeros in the places shown in $\boldsymbol{K}$ of eqn (4), which is the condition for the laminate to have orthotropic symmetry. If the distribution of $|\theta_i|$ is not symmetric about the middle plane of the laminate, the theory used here would be incomplete since the plate will exhibit bending–stretching coupling. If the angles satisfy this condition but are not actually symmetric, the matrix may contain additional stiffness terms in place of the four zeros, representing additional bending–twisting coupling. (The model used here already includes the bending–twisting coupling which arises with orthotropic plates: departure from orthotropic symmetry introduces extra effects of this nature.) The theory used here could be readily extended to include this case.

It is straightforward to obtain the damping properties of the laminate. The damping is assumed "small", i.e. modal damping factors small compared with unity, or Q-factors $\gg$ 1. (Q-factors are the inverse of modal loss factors: $1/Q$ is the fractional loss of energy per radian in a free vibration.) In accordance with the viscoelastic correspondence principle[2,3], the elastic constants are made complex by introducing damping constants η_1, η_2, η_3 and η_4 so that D_j is replaced by $D_j(1 + i\eta_j)$, j = 1,2,3,4. Taking the imaginary part of eqn (5), we deduce that

$$\eta_j^{\mathrm{lam}}=\frac{4}{n^3D_j^{\mathrm{lam}}}\sum_{i=1}^{n}\alpha_i\mathrm{Im}\{D_j^i\} \; j=1,2,3,4 \quad (6)$$

The laminate damping constants thus follow directly from standard laminate theory in a similar fashion to the elastic constants. Note that these damping "constants" will in general be frequency-dependent[8,9], but empirically the dependence is usually found to be rather weak at low audio frequencies as are relevant here.

This procedure for obtaining laminate properties from ply properties can also be inverted: sometimes, the most reliable values for ply properties might be obtained by measuring properties of a known laminate and working backwards. This is achieved as follows. Performing the matrix multiplication of eqn (3) gives four equations for the elements of the global laminate stiffness matrix. Substituting these into eqn (5) gives expressions for the laminate elastic constants which can then be rearranged to give four simultaneous equations which may be written:

$$A\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = \frac{n^3}{4}\begin{bmatrix} D_1^{\rm lam} \\ D_2^{\rm lam} \\ D_3^{\rm lam} \\ D_4^{\rm lam} \end{bmatrix} \tag{7}$$

where

$$A = \begin{bmatrix} p & q & r & q \\ 2q & p+r & 2q & -2q \\ r & q & p & q \\ 4q & -4q & 4q & s \end{bmatrix} \tag{8}$$

with

$$p = \sum_{i=1}^{n} \alpha_i c_i^4 \quad q = \sum_{i=1}^{n} \alpha_i s_i^2 c_i^2 \quad r = \sum_{i=1}^{n} \alpha_i s_i^4$$

$$s = \sum_{i=1}^{n} \alpha_i (s_i^2 - c_i^2)^2$$

and s_i, c_i as in eqn (4) and α_i as in eqn (5). The ply elastic constants may then be obtained straightforwardly by inverting A (provided that it turns out to be non-singular). Exactly the same procedure may be followed to obtain the ply damping constants, which satisfy the equation

$$A\begin{bmatrix} D_1\eta_1 \\ D_2\eta_2 \\ D_3\eta_3 \\ D_4\eta_3 \end{bmatrix} = \frac{n^3}{4}\begin{bmatrix} D_1^{\rm lam}\eta_1^{\rm lam} \\ D_2^{\rm lam}\eta_2^{\rm lam} \\ D_3^{\rm lam}\eta_3^{\rm lam} \\ D_4^{\rm lam}\eta_4^{\rm lam} \end{bmatrix} \tag{9}$$

with the same matrix A as in eqn (8).

Once a particular vibration mode and its frequency are known the modal Q-factor can be determined in terms of these laminate damping constants. (Modes involving different types of deformation of the plate can be expected to have different Q-factors.) The Rayleigh quotient derived from the expression (1) and the corresponding expression for the kinetic energy of the plate involves the sum of the four quantities

$$J_1 = \frac{D_1 h^2 \iint w_{xx}^2\, dA}{\omega^2 \rho \iint w^2\, dA} \quad J_2 = \frac{D_2 h^2 \iint w_{xx} w_{yy}\, dA}{\omega^2 \rho \iint w^2\, dA}$$

$$J_3 = \frac{D_3 h^2 \iint w_{yy}^2\, dA}{\omega^2 \rho \iint w^2\, dA} \quad J_4 \frac{D_4 h^2 \iint w_{xy}^2\, dA}{\omega^2 \rho \iint w^2\, dA} \tag{10}$$

where ρ is the mean density of the laminate and ω is the frequency of the mode in question[1,4]. These Js are dimensionless constants which quantify the fraction of strain energy associated with each of the elastic constants D_j. They are readily computed from the elastic mode shape. They are normalised so that $\Sigma J_i = 1$.

Now when the elastic constants are replaced with the complex moduli, Rayleigh's principle shows that the imaginary part of the frequency, and thus the modal damping factor, can be calculated to good approximation using the elastic mode shapes[1,4]. It follows that

$$Q^{-1} \approx \eta_1^{\rm lam} J_1 + \eta_2^{\rm lam} J_2 + \eta_3^{\rm lam} J_3 + \eta_4^{\rm lam} J_4 \tag{11}$$

This quantifies the intuitive notion that the total energy dissipation rate is a weighted sum of the different loss factors based on the partitioning of strain energy among the corresponding elastic moduli.

EXPERIMENTAL METHOD

The laminates used for experimental testing of the above theory consist of several plies of equal thickness, of a uni-directional, carbon-fibre "pre-preg" in an epoxy resin matrix (2224 carbon-epoxy). The lay-up details are such that the laminates are specially orthotropic; the angles θ_i were as follows:

Laminate 1: six, 0° plies
Laminate 2: 0°, 90°, 0°, 0°, 90°, 0°
Laminate 3: +45°, −45°, −45°, 0°, +45°, +45°, −45°

Laminate 1 is simply a uni-directional laminate from which the elastic and damping constants of a single ply could be determined directly.

The properties of each laminate have been determined using a method introduced by McIntyre and Woodhouse[1] and described in some detail in that reference. The method is based on measuring the frequencies and Q-factors of the first few vibration modes of rectangular plates with free boundaries. (These modes involve both bending and twisting motion—unless Poisson's ratio is zero it is not possible for a free plate to vibrate with pure flexural or pure torsional motion[10].) By fitting these frequencies and Q-factors to the pattern predicted by thin-plate theory, the values of $D_i^{\rm lam}$ and $\eta_i^{\rm lam}$ may be determined.

Frequencies were measured using sinusoidal driving to excite Chladni patterns, in which the nodal lines are revealed by a powder such as tea leaves. Damping was measured from the temporal decay characteristics following impulsive excitation. For this purpose, the vibration was measured using a very small accelerometer (Brüel and Kjaer type 8307, weighing under 1 g). This must be attached to the plate near, but not on, a nodal line of the mode being observed, to minimise mass loading and additional damping due to vibration of the accelerometer's cable. The free decay rate was determined by digitising the transient vibration signal in a data-logger, performing time-frequency analysis on it[11], and best-fitting an exponential decay rate to the time-varying amplitude of the relevant spectral peak (as described in ref. 1).

Great care was taken to support the plates only at nodal points to preserve the "free" boundary conditions. For the Chladni patterns it is sufficient to support the plate on very small pieces of soft foam positioned beneath nodal lines, but for damping measurements more care is needed. For this study, points on the plates were identified at which nodal lines for different modes intersect, and small holes were drilled there. To measure the damping, the plate was suspended by fine thread through a hole relevant to the mode in question. The measurement accuracy is of the order of 1% for frequencies and 10% for Q-factors—the damping is much more sensitive to disturbance by the supports and the instrumentation used to observe the vibration, and modes with very low damping are particularly difficult to measure reliably. To put these error bounds in context, for vibration engineering applications it is common to require accurate resonant frequencies, but it is rare to know the damping to better than a factor of two because of the additional effects of damping at joints, which are usually very hard to model.

RESULTS AND DISCUSSION

A representative set of results is shown in *Table 1*, for a particular (nearly square) plate of laminate 2. Results for the first seven vibration modes are shown. Alongside sketches of the mode shapes, the measured frequencies and Q-factors are compared with those resulting from a best fit to thin-plate theory. The procedure for this fitting is described in detail in ref. 1. There are two stages: the modes 1, 2 and 4 in *Table 1* consist, to a first approximation, of pure twisting, bending along the y-axis and bending along the x-axis respectively, and they allow first estimates to be calculated for D_4, D_3 and D_1 respectively, and for the corresponding damping constants. These first estimates are then adjusted to give the final answers by using computed results and perturbation theory.

To complete the determination of the elastic and damping parameters of this plate, further measurements were taken after the aspect ratio of the plate was adjusted. By choosing the correct aspect ratio the plate becomes "effectively square", in the sense that the fundamental bending resonance frequencies "across the plate" and "along the plate" would be equal in the absence of Poisson coupling. Under these conditions the influence of the Poisson's ratio

Table 1 Results from a plate of dimensions 178 × 176 × 0.95 mm made of Laminate 2[a]

Mode	Sketch	Measured frequency (Hz)	Best fit to theory	Measured Q	Best fit to theory
1		53	52	90	86
2		131	131	340	333
3		167	167	146	158
4		200	200	620	620
5		223	225	270	271
6		311	318	146	163
7		361	361	360	333

[a]"Best fit" results were computed using the elastic and damping constants listed under "Laminate 2, measured" in *Tables 4* and *5*

Table 2 Results from a plate of dimensions 126 × 108 × 0.92 mm made of Laminate 1[a]

Mode	Sketch	Measured frequency (Hz)	Best fit to theory	Measured Q	Best fit to theory
1		113	112	84	79
2		176	176	100	97
3		289	288	76	87
4		468	468	515	517
5		478?	482	?	102
6		511	521	270	268

[a]"Best fit" results were computed using the elastic and damping constants listed under "Laminate 1, measured" in *Tables 4* and *5*. The Q-factor of mode 5 could not be determined because of interference from mode 4

Table 3 Results from a plate of dimensions 230 × 230 × 1.07 mm made of Laminate 3[a]

Mode	Sketch	Measured frequency (Hz)	Best fit to theory	Measured Q	Best fit to theory
1		55	55	90	89
2		81	81	600	578
3		101	102	300	309
4/5		173	176	270?	323
6/7		230	237	250?	252
8		299	309	255	273
9		345	351	500	491
10		365	364	390	370

[a]"Best fit" results were computed using the elastic and damping constants listed under "Laminate 3, measured" in *Tables 4* and *5*. The Q-factors of modes 4/5 and 6/7 could not be determined accurately because of beating between the close pairs of modes

term D_2^{lam} on both frequency and damping is maximised. The elastic modulus can then be measured accurately, as described in ref. 1. The corresponding damping term η_2^{lam} may or may not be possible to determine accurately—for some materials it has been found that η_2^{lam} is too small to be measured reliably, even by this optimal approach. For this particular lay-up of laminate, the elastic modulus D_2^{lam} was found to be zero to the accuracy of the measurements. It follows that $D_2^{\text{lam}}\eta_2^{\text{lam}}$ is zero, so that η_2^{lam} cannot be determined.

Table 1 shows that the measured and fitted frequencies match very closely. The agreement of the Q-factors is less close, but is acceptable given the lower accuracy of the measurements. Note that the anisotropy of the laminate material leads to a substantial variation in modal damping, in this case by a factor of 7 (between mode 1 and mode 4). This is the variation which is quantified by eqn (11) once the values of the damping constants η_i^{lam} are known. It is quite clear that the pattern of variation of the modal damping factors is well captured by the theory (which is, recall, the simplest thin-plate theory neglecting shear and rotatory inertia).

Table 4 Results for elastic constants

Laminate	D_1^{lam} (GPa)	D_2^{lam} (GPa)	D_3^{lam} (GPa)	D_4^{lam} (GPa)
1, Measured	7.07	0.27	0.54	1.12
1, Deduced from Lam 2	6.41	0	0.45	1.16
2, Measured	4.86	0	1.99	1.16
2, Deduced from Lam 1	5.38	0.27	2.23	1.12
3, Measured	2.1	2.9	2.1	7.1
3, Deduced from Lam 1	2.26	3.37	2.24	7.32
3, Deduced from Lam 2	2.08	2.97	2.07	6.57

"Measured": the results of best-fitting measurements
"Deduced from Lam 1": laminate theory based on ply properties from the first row
"Deduced from Lam 2": laminate theory based on ply properties obtained via inverse laminate theory using results from row 3

Table 5 Results for damping constants

Laminate	η_1^{lam} ($\times 10^{-3}$)	η_2^{lam} ($\times 10^{-3}$)	η_3^{lam} ($\times 10^{-3}$)	η_4^{lam} ($\times 10^{-3}$)
1, Measured	1.9	− 22.7	10.1	12.9
1, Deduced from Lam 2	1.4	0	11.2	12.0
2, Measured	1.6	–	3.0	12.0
2, Deduced from Lam 1	2.1	− 22.7	3.4	12.9
3, Measured	3.8	0	3.8	1.6
3, Deduced from Lam 1	3.0	− 0.3	3.0	3.4
3, Deduced from Lam 2	2.7	− 0.8	2.8	2.9

"Measured": the results of best-fitting measurements
"Deduced from Lam 1": laminate theory based on ply properties from the first row
"Deduced from Lam 2": laminate theory based on ply properties obtained via inverse laminate theory using results from row 3

A similar level of agreement for both frequencies and Q-factors was obtained for the plates of laminates 1 and 3, reproduced in *Table 2* and *Table 3*. The tested plate of laminate 1 was rather small, because the unidirectional material proved rather fragile, cracking readily along the fibre direction. The plate of laminate 3 had the correct proportions to be "effectively square", as explained above. (In this case it was also physically square, as will be commented on below.) This gives it a characteristically different series of mode shapes.

The full set of results for elastic constants D_i^{lam} is summarised in *Table 4*, and those for damping constants η_i^{lam} in *Table 5*. The unmeasurable η_2^{lam} for laminate 2 is shown as a dash. For both sets of results, the measured values are compared with predictions of laminate theory. For Laminates 2 and 3, values are compared with predictions based on ply properties deduced directly from the values of Laminate 1, the unidirectional material.

For Laminates 1 and 3, a second comparison is also made. In practice, it may be inconvenient or impractical to manufacture unidirectional material for the purpose of calibration measurements of the ply properties. It may be easier, and also perhaps more reliable, to measure the properties of a known laminate, use inverse laminate theory (eqns (8) and (9)) to deduce ply properties, then re-use these values in normal laminate theory to predict the properties of other lay-ups of the same ply material. This procedure has been carried through here, both for elastic and damping constants. The results for Laminate 2 have been inverted to produce ply properties which are listed in the second row of *Tables 4* and *5*, and these used to predict values for Laminate 3 which are shown in the final row of each table. Note that for this calculation, η_2^{lam} for laminate 2 has been put equal to zero.

The general level of agreement revealed by these tables is reasonably satisfactory: laminate theory comes within about 3% for the elastic constants, and usually within about 30% for damping constants. (Recall the earlier comment on the relative crudeness of the damping estimates needed for typical vibration engineering applications.) Part of the blame for the errors in predictions here is associated with uncertainty about the plate thicknesses. The manufacturing method produced plates with one face smooth and the other slightly textured. Thickness measurements quoted were made with a flat-footed micrometer (and are the average of five positions in each case). The elastic properties quoted here depend on the fourth power of thickness.

Ply properties can be determined successfully either from unidirectional material or by using inverse laminate theory on a known laminate. There seems to be little to choose in terms of accuracy between the two methods, and the choice in practice would be governed by convenience.

There are some deviations from this level of agreement which deserve discussion. First, as was noted above, η_2^{lam} is often hard to determine with accuracy. The determining factor is the value of J_2 for the modes in question. (For some typical sets of values of the J_i, see ref. 1, Tables 3, 6–9.) For the plates tested here Laminate 2 gave generally small values, while Laminate 3 gave much larger values (because it was "effectively square"). This means that the disparity of values of η_2^{lam} relating to Laminate 2 is not surprising, whereas the good agreement between all methods for Laminate 3 is a genuine (and quite sensitive) test of the theory, even though the numerical value of η_2^{lam} turns out to be very small for this material.

Otherwise, the only glaring disparity revealed in *Tables 4* and *5* is in the value of η_4^{lam} for Laminate 3. The measured value is about one-half the value predicted by either application of laminate theory. We have no ready explanation for this—shear deformation (i.e. twisting motion) of this particular laminate really does appear to be less lossy than one would expect on the basis of the other laminates, beyond the uncertainty attributable to experimental error. More extensive testing would be needed to see if the problem is reproducible, or whether it is an artefact of detailed manufacturing processes for these particular samples.

The final point worthy of comment is that Laminate 2 can be seen from *Table 4* to be quasi-isotropic: $D_1^{lam} \approx D_3^{lam}$ so that the bending stiffnesses in the two principal directions are equal. However, this plate is, in fact, by no means isotropic. If one deduces a value of Poisson's ratio from the value of D_2^{lam}/D_1^{lam} it turns out to be about 0.7, and this in turn would predict a value $D_4^{lam} = 1.2$ GPa, which differs from the true value by more than a factor of 5.

CONCLUSIONS

It has been verified that thin-plate bending theory can be used with confidence to predict the low-frequency vibration behaviour of free-edged, CFRP laminates and that laminate theory is capable of predicting elastic behaviour with sufficient accuracy to enable frequency and mode shape predictions to be made. It has also been shown that laminate theory can be extended to predict damping properties. Damping constants are predicted less accurately by laminate theory than are elastic constants, but an accuracy level of 30% is in fact adequate for most engineering purposes.

Results have been presented in terms of an unfamiliar combination of elastic constants, introduced in an earlier study[1] and defined in eqns (1) and (2). These constants are a convenient set to use when discussing thin-plate deformation or vibration. They are also the constants which occur naturally in the expression for the strain energy in terms of the plate centre-plane displacement, so that they lend themselves to use in Finite-Element computations.

It has been demonstrated that the elastic and damping properties of the plies of a laminate can be deduced from measurements on the complete laminate, using inverse laminate theory. This approach might have advantages in some circumstances: it allows the necessary parameter values to be deduced from a standard "production" laminate, and it obviates the need to make special unidirectional laminates for calibration purposes.

Elastic constants and, particularly, damping constants are likely to be sensitive to variations of manufacture. For example, incomplete bonding between plies is likely to lead to reduced stiffness and greatly increased damping. The test procedures employed in this study, based on simple observation of vibration frequencies and damping factors of rectangular plates, might form a basis for non-destructive testing for quality control.

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