Body Vibration of the Violin—What Can a Maker Expect to Control?

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ABSTRACT
At low frequencies it is sensible to describe violin body vibration in terms of individual modes, and for a maker to seek to control these modes explicitly. At higher frequencies this ceases to be a realistic goal. The modes overlap in frequency and are very sensitive to small changes in the construction. The acoustical information useful to a maker then relates to controllable features of the behavior that "shine through" the complexity of detail.

INTRODUCTION

The constructional details of a violin body control its vibration behavior, which in turn controls the sound and playing properties of the instrument. This seems obvious, yet the detailed links along the chain have proved remarkably hard to elucidate. Although considerable research effort has gone into violin acoustics (see particularly references [1, 2]), it is rarely possible to give satisfactory answers to the questions posed by instrument makers, who usually want to know how the "sound" or "playing quality" will be affected by a particular structural change.

A violin, in common with any other structure undergoing small-amplitude vibration, can be characterized by its set of vibration modes. Each mode has a resonant frequency, a damping factor, a mode shape and a radiation efficiency and pattern, and once you know this information about each mode the behavior of the instrument is fully described. In the audible range of frequencies a violin has several hundred vibration modes. In some sense, the task of the violin maker is to control the parameters of these modes to produce the desired vibration behavior. However, a maker cannot expect to be able to manipulate more than a tiny fraction of these parameters explicitly. Understanding which features can in practice be controlled is an important task for violin acoustics, and is the subject of this article. The question will not be answered fully, of course, but some light can be shed by reviewing what is known in the context of the contemporary approach to other complex vibration problems.

Any structural change may influence the modal parameters. A change of mode shape may alter the radiation efficiency, but perhaps more significantly it may change the mode amplitude at the bridge and hence the efficiency of excitation by the string. Particularly if a mode has a nodal line that passes close to the bridge, small movements of that nodal line may have large effects on strength of coupling to the string. If the mode plays a significant role in the radiated sound, then changes in its frequency and damping factor are also likely to produce audible consequences.

Altering the radiation of sound is not, however, the only route by which a structural change may have musical consequences. As well as being interested in "sound", a player is also interested in various aspects of "playability". One instrument, or string, or note, may be found "easier to play" than another, and such differences can be very important. Any influence of the body vibration on the response of the string can only come through the bridge (or perhaps through vibration of the fingerboard at the other end of the vibrating length of string). To a first approximation, we should expect all variations in playability between instruments to be attributable, somehow, to differences in the driving-point response at the string notch [3, 4]. This depends, again, on the modal amplitudes at that point, but this time weakly-radiating modes might be just as important as strongly-radiating ones.

In their very different ways, important contributions to this subject have been made by Cremer [1], Hutchins (see for example the section introductions and reprinted papers in [2]), Weinreich [e.g. 5, 6], Marshall [7] and Bissinger [e.g. 8, 9], among others. But no one has contributed more to this area than Jansson, whose entire research career has been focused on the vibration of the violin body and its enclosed air. His contributions include many measurements, especially via laser holography [10] or bridge admittance [e.g. 11], systematic investigations of the effect of various structural changes in his collaboration with the violin-maker Niewczyk [e.g. 12], and recently his recognition of a significant feature of instrument response originally called the "bridge hill [e.g. 13], to which we return later.

OVERLAP FACTORS

Modal Overlap Factor

One way to explore the extent to which a maker can control the vibration modes of an instrument is through the interplay of two quantities, the modal overlap factor and the statistical overlap factor. The modal overlap factor is the simpler to understand: it is the ratio of the bandwidth of individual resonance peaks to the average spacing of adjacent resonances [e.g. 14]. When modal overlap is low, each mode contributes a recognizable peak to any frequency response function. When modal overlap is high, on the other hand, several modes make a significant contribution at a given frequency, and the total response is governed by a summation of these contributions. The result will depend on the relative phases and amplitudes of the various modes. Peaks in a frequency response function will be governed by these interference effects, not by individual modes. It is useful to think about individual modes at low modal overlap, but with high modal overlap it is likely to be more appropriate to use a statistical description, dealing in such quantities as average levels, average peak spacings, and typical peak-to-valley heights. The most familiar example occurs in room acoustics. Modal overlap at normal audio frequencies in a moderate or large room is very high, and room acoustics is a statistical science [e.g. 15].

Statistical Overlap Factor

The statistical overlap factor is defined in a similar way to the modal overlap factor, to characterize the sensitivity of mode frequencies to structural changes. For example, a luthier will ordinarily only depart from accepted patterns of thickness distribution in the top and back plate of a violin by fractions of a millimeter. For a given instrument, we can imagine randomly changing the thickness distributions within that tolerance. Each mode frequency will change a little, and the sensitivity of the structure can be characterized by the range of these frequency shifts when many different thickness perturbations within the assumed tolerance are tested. The statistical overlap factor is defined as the ratio of this average shift to mean modal spacing [16].

Different statistical overlap factors can be defined for different populations of violins. What has just been described is the factor relating to "all respectable violins based on a single model" since only graduation changes were considered. A different answer would probably be obtained if one considered the population of "all respectable violins", in which outline and arching were also varied. A different answer again would be obtained from instruments deliberately built to be as similar as possible in all respects. One could even consider the same violin, measured at different times and therefore under different conditions of humidity, temperature and recent playing history.

If the statistical overlap factor is small, individual modes more or less retain their identity under the permitted variation in the structural properties. If statistical overlap is high, on the other hand, individual modes will have very little recognizable identity from one instrument to another in the chosen population. A variation within the acceptable limits moves modes far enough that they interact strongly with their neighbors. The number of modes remains the same, but individual mode shapes are likely to change beyond recognition. With high statistical overlap it makes little sense to look for "the same mode" in two different instruments: any scheme for classifying and labeling modes breaks down under these conditions.

Both modal and statistical overlap factors tend to increase with frequency. The modal spacing in a violin box is approximately constant at low frequencies (governed by the bending-plate behavior of the box) and then decreases at higher frequencies (from the increasing contribution of internal air modes) [6]. Modal damping factors remain approximately constant with frequency, so that the half-power bandwidth increases roughly linearly with frequency. It follows that the modal overlap, which is low at low frequencies, grows with frequency and eventually becomes large. In a similar way, the effect of a variation in thickness, arching shape or material properties is to shift modal frequencies by a certain percentage, which is roughly independent of frequency. The absolute fluctuations in modal frequency thus increase approximately linearly with frequency, and the statistical overlap factor does the same. Combining these two effects, we should expect a deterministic description of individual modes to be useful at low frequencies, but for sufficiently high frequencies a different, statistical, description needs to take over [see e.g. 17].

"... a maker cannot expect to manipulate more than a tiny fraction of these parameters"

Experimental Values

To find out what precisely is meant by "sufficiently high frequencies" requires experimental data. If a single measurement is to be used, then the best candidate is the input admittance (or mobility) at the bridge top, in the plane of bowing. This contains the main information relevant to any differences in playing behavior. It is not directly a measurement of radiated sound, but it does give some information on that subject while circumventing the difficulties associated with radiation patterns and room acoustics, which add additional features to the response and make interpretation more difficult. The input admittance governs the rate of energy transfer from the vibrating string into the body. This energy is either lost in internal damping in the body, or is radiated as sound. If all modes had the same radiation efficiency the energy radiated would be a fixed fraction of this total input energy and would thus follow the same response curve. The specific measurement used here is of the velocity response at the E-string corner of the bridge in response to a force applied at the G-string corner, both in the direction of bowing. This measurement gives a good approximation to the input admittance (or mobility) at the bridge top [11].

Figure 1 shows a typical result for a good modern violin, and gives information about modal overlap. Figure 2 shows superimposed results for five different violins by the same maker (including the instrument shown in Fig. 1), and gives information about statistical overlap. Up to the deep antiresonance around 650 Hz, all the curves are rather similar and show quite well separated peaks. Both overlap factors are fairly small in this range. At slightly higher frequencies, up to 1.5 kHz or so, the curve of Fig. 1 shows peaks that are beginning to overlap. At the same time, the differences between the set of curves in Fig. 2 increase. We can deduce that both the modal overlap factor and the statistical overlap factor are reaching significant values in this range. At frequencies above 1.5 kHz the general trend of the five curves remains similar, but they diverge considerably in detail. Both overlap factors are now greater than unity, and on both counts any attempt to describe the behavior in terms of individual modes is likely to be extremely difficult and not very illuminating.

When a comparison is made of input admittances of any set of normal violins of reasonable quality, a similar description is generally found to apply: see for example the results published by Jansson [13] and Dünnwald [18]. The conclusion is that up to about 650 Hz any two violins are likely to show mode shapes that are recognizably related. For frequencies above that up to 1 kHz or so it is possible with careful measurements [e.g. 7, 9] to determine the modes despite moderate overlap, but the shapes are likely to be much less recognizable from one instrument to another. At higher frequencies it is doubtful whether there is any virtue in talking about individual modes, because both the modal overlap factor and the statistical overlap factor are probably too high.

THE LOW-FREQUENCY MODES

The modes below 650 Hz or so, common to most violins, are familiar to CAS readers.

(i) "Plate Modes"

There are three modes in this frequency range primarily arising from bending and stretching motion of the top and back plates of the box. The modes typically cluster in the range 380–600 Hz, and some aspect of this cluster of modes corresponds to what was called the "main body resonance" in the earlier literature of violin acoustics [e.g. 19]. Examples of these three modes are shown in Fig. 3, for one of the violins whose behavior was shown in Fig. 2. Pictures of similar modes have been shown by many authors, e.g. Marshall [7], Jansson et al. [10].

The lowest of the three, labeled by Jansson "C2" and by Marshall "vertical translation of C-bouts", is shown in Fig. 3a. It typically occurs in the frequency range 380–440 Hz. It can be thought of as a mode in which the entire box behaves rather like a thick plate, back and top moving approximately together at each point in a twisting deformation with roughly one complete wavelength of twist in the length of the body. The other two modes, shown in Fig. 3b,c, are "twins", labeled by Bissinger "B1–" and B1+" [20]. (Jansson and coworkers [e.g. 11] label them "T1" and "C3" respectively.) The most graphic name for them is the "baseball modes", since in each case there is a single, sinuous node line going around the body like the seam on a baseball. B1– typically occurs in the range 450–480 Hz, B1+ in the range 530–570 Hz. Unlike C2, these two modes involve significant volume change of the box, and they are thus strong radiators of sound.

(ii) "Air Modes"

There are two low-frequency modes associated primarily with air-pressure variation in the internal cavity of the violin: a modified Helmholtz resonance typically around 280 Hz, usually denoted "A0"; and a first standing wave in the length of the box similar to an organ-pipe mode, typically in the range 470–490 Hz, first identified by Jansson [21] and denoted by him "A1". Figure 3d illustrates the

Figure 1. Input admittance at the bridge, in the direction of bowing, for a violin by David J Rubio.

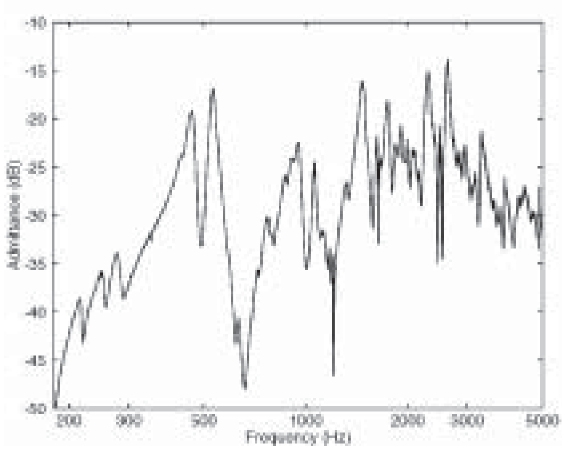


Figure 2. Input admittance at the bridge, in the direction of bowing, for five violins by David J Rubio, including the one shown in Fig. 1.

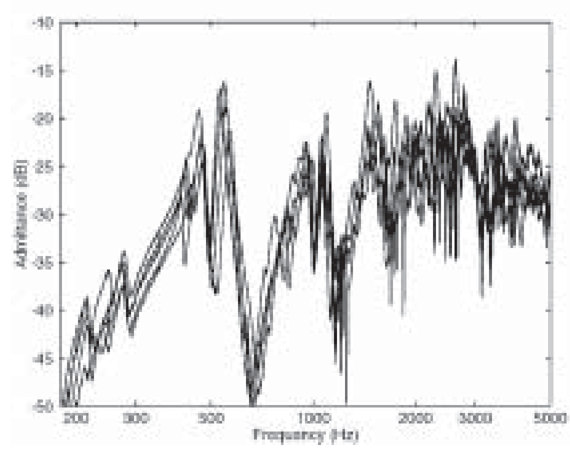
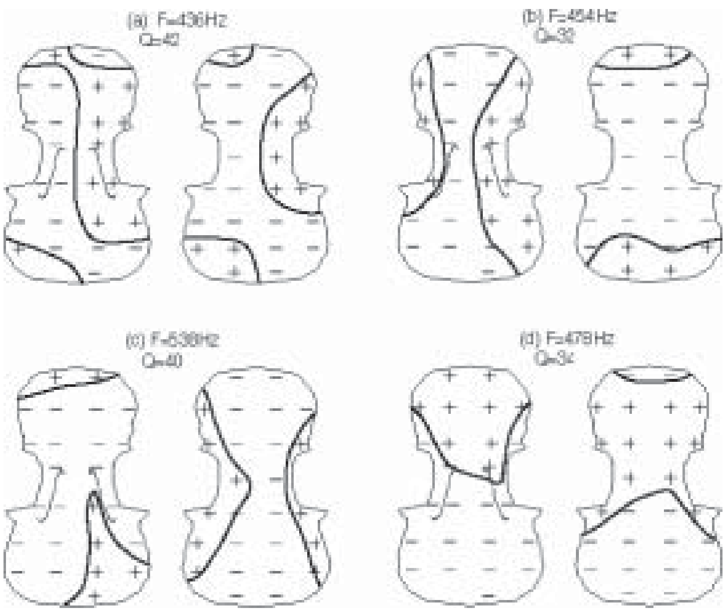


Figure 3. Mode shapes for some significant low-frequency modes of a typical violin: (a) C2; (b) B1-; (c) B1+; (d) A1. Node lines are shown as heavy lines. Back and front plates are viewed from the outside; + and - signs indicate the positions of data points and the relative phase of motion, + denoting "outwards" and - denoting "inwards". Frequencies and Q factors are also shown.



mode A1. This is primarily an air mode, but with a careful measurement it shows up through the response of the structure, the upper and lower bouts alternately "inflating" and "deflating" in response to the pressure changes inside.

(iii) "Tailpiece Modes"

At low frequencies, the tailpiece behaves as a rigid body suspended on the strings and tailgut. As a rigid body it would have six degrees of freedom, but for one of these (axial motion parallel to line of strings and tailgut) the suspension is stiff, pushing the associated resonant frequency out of the low-frequency range. There remain five tailpiece resonances to be considered. Stough [22] has studied them in some detail. He reports that three typically occur below 200 Hz, below the fundamental of the open G string (196 Hz), while the remaining two can range widely in the range 300–800 Hz, depending on tailpiece mass and tailgut length.

(iv) "Neck/Fingerboard Modes"

Finally, there is a group of modes based on bending or twisting beam-like behavior of the neck and fingerboard, with the attached box acting as an extension of the beam. The list given here is not exhaustive: we discuss only the two modes which fall in the range 200–700 Hz and which seem to be of some importance because they can interact significantly with other modes already listed. One mode, usually called "B0", typically occurs at a frequency in the vicinity of the lowest "air" mode, A0. It involves motion in which the scroll, neck and body vibrate as a single beam in its lowest free-free mode, while the cantilever-projecting length of the fingerboard vibrates vigorously in the opposite phase to the body beneath it. The second mode of relevance here is the lowest torsional mode of the projecting portion of the fingerboard. This mode often occurs at a frequency within the "main body cluster" discussed in (i) above.

Of these low-frequency modes, measurements suggest that only A0, B1– and B1+ are strong radiators of sound in the violin, although Bissinger has shown that A1 can be a significant radiator of sound in larger instruments of the violin family [23]. The tailpiece and neck/fingerboard modes radiate very little, although their effect may be heard clearly by the player, who has one ear very close to the violin body. All these modes might in principle contribute to issues of "playability" by influencing the driving-point response at the bridge. They might also influence behavior by acting as "tuned absorbers", increasing the energy dissipation of the violin near their resonant frequency in exactly the same way as a "wolf suppressor" [24].

Although the items in this list have been described as "modes", this can be misleading even at these low frequencies. The descriptions above are of a set of types of motion of the violin body and enclosed air, which often appear as separate mode shapes. However, strictly this is a description of the degrees of freedom of the violin body which taken in isolation would have resonant frequencies in this frequency range. If two or more of these would have resonant frequencies that are close together, then the actual modes of the violin will show combinations of the motions described above. The particular case of coupling of the motions described above as the A0 and B0 "modes" has been studied in detail by Woodhouse [25], and illustrated with experimental results. A similar analysis could be developed for other combinations. In every case, one should expect to see "veering" behavior [25, 26]. If adjustments are made which would tend to bring two frequencies close together, then strong interaction will occur and the two frequencies will in fact never approach more closely than a certain minimum separation.

As the two overlap factors increase, so the likelihood of such interactions increases. Even in the low-frequency range interactions are quite common, and it is not always easy to recognize all the members of this "canonical" set of modes on a given instrument. The situation becomes rapidly more complicated at higher frequencies. It is probably safe to say that the modes listed above are the only ones which usually retain their identity sufficiently clearly that it makes sense for an instrument maker to think in terms of explicitly controlling their frequencies, damping factors and mode shapes by structural adjustments. An effort to do just that has been described by Schleske [27], in his procedure for making a "tonal copy" of an instrument.

WHAT CAN BE CONTROLLED AT HIGHER FREQUENCIES?

At higher frequencies, things are not so simple. The modes overlap and blur together, and the variability of mode shapes between instruments is much greater. Although, mathematically speaking, the modes still determine the behavior, this point of view becomes progressively less useful to the maker. However, the fact that a maker cannot control everything at higher frequencies does not mean that they cannot control anything. A natural approach is to look for aspects of the violin that might retain deterministic behavior at higher frequencies and thus offer control to the maker in a straightforward way. The list that follows is not by any means exhaustive, but it is intended to indicate some possibilities and to encourage the search for others.

Material Properties

The first item to consider does not concern constructional details but material choice and behavior. The physical properties of the wood, including any modifications due to seasoning, chemical treatment and/or varnishing system, obviously have a strong influence on vibration behavior. To an extent, variations in wood properties can be compensated by varying the constructional details such as graduation: this is the rationale for plate-tuning procedures. However, this compensation cannot by any means be complete. If there is any substance in the persistent stories of "old instruments being better" or of "magic varnish", it must surely be sought in this area.

The relevant properties of wood are density, several types of stiffness, and associated measures of damping. The three most important stiffnesses can be visualized easily in terms of flexing a thin plate of wood, such as a guitar top plate blank. They relate to bending along the grain direction, bending across the grain direction, and twisting. These are influenced by the growth of the particular tree, and also by the cutting: for example, wood with "runout" will have a reduced long-grain stiffness, and wood not cut accurately on the quarter will have a reduced cross-grain stiffness [28].

The damping properties are likely to vary with frequency. This fact offers the most plausible explanation for old wood behaving differently from newer wood. Old wood will have changed. Chemical changes occur with drying, loss of other volatile components, oxidation, and reactions with atmospheric pollutants. Chemical changes may also be deliberately made, for example when ozone, ammonia or nitric acid is used to treat wood for one reason or another. Physical changes will also occur in the wood, especially in wood which has been in an instrument for many years being vibrated. Cells in the wood structure may separate along their joins, or micro-cracks may form in the individual cell walls. All these changes will modify the material properties. In particular, it seems a good guess that such damaged wood will have higher damping, especially at the higher frequencies. If damping increases at higher frequencies, this would have a "filtering" effect on the sound of the instrument, similar to turning down the treble control on a hi-fi system. There is some evidence of such an effect in acoustic measurements on older instruments [18], but there is very little reliable data on the actual changes to wood/varnish properties from these various effects. There is scope for fruitful research in this area.

The Bridge

Next, we turn to parts of the violin structure which are likely to have well-defined and controllable vibration resonances up at frequencies where the violin as a whole has significant modal overlap. The first and most obvious candidate is the bridge. The vibrating strings apply forces to the top of the bridge, and the bridge feet transfer these forces to the top plate and hence into body vibrations which can radiate sound. Since all vibration has to pass through the bridge, it will be modified by the vibration response of the bridge itself. A typical modern violin bridge has two important resonances at moderate audio frequencies: one around 2–3 kHz in which the bridge top moves from side to side by "bending at the waist", and another around 6 kHz in which the top moves vertically by "bending at the knees" [1]. These resonances can be moved significantly by relatively small changes in the carving of the bridge — this is the main source of the well-known influence of bridge cutting on sound.

Jansson has drawn attention to a feature of the input admittance curves of many good violins, a broad hump in the response in a frequency range typically 2.5 kHz [e.g. 13]. He originally attributed this to the filtering effect of the lower bridge resonance just mentioned, and named the feature the "bridge hill". (Figure 1 does not show this feature very clearly, but it can be seen in Fig. 2 in the average behavior of the plotted curves in the range 2–3 kHz.) Certainly, a broad hump of this kind is just what one would expect the bridge resonance to create. However, more recent work [29] has indicated that other features as well as the bridge influence the "hill", and at present the precise details are not clear. What is clear is that the "hill" is an example of the kind of feature we are interested in here, a high-frequency feature that somehow shows through the quasi-random peaks and dips caused by the overlapping modes.

The "Island"

Another candidate for influencing Jansson's "hill" is a feature of the violin that is slightly less obvious than the bridge. The slot-like shape of the f-holes creates an approximately square region of the top plate that is somewhat isolated from the rest of the instrument. This region was called "the island" by Cremer, and was included as a separate element in his pioneering attempt to compute body modes of the violin from a theoretical model [1]. His idea was that this portion of the top plate may be sufficiently isolated as to have a recognizable and distinctive influence on the vibration, particularly since the bridge feet sit in this region. One might perhaps expect a small number of local "resonances" of this island to occur which, when coupled to the rest of the instrument structure, produce similar features to the "bridge hill". This is still an unresolved question, but it deserves further study.

The Soundpost

The soundpost is usually regarded as a rigid link, serving to couple the top and back plates. However, as a thin rod, it will have resonances of its own. The lowest would be a bending resonance with approximately a half-wavelength in the length of the post. The frequency will depend on the length, diameter and material properties of the post, and taking typical values of these parameters (spruce, length 60 mm, diameter 6 mm) one might expect this to fall in the vicinity of 3 kHz. This could be expected to have some influence on the vibration behavior in this frequency range, but some theoretical modeling is needed to determine exactly what form it would take.

One interesting point to note concerns the accuracy of fit of the ends of the soundpost. Good soundpost fit, and the right degree of tightness, are well known to be important. It has also been suggested that shaping the post to change its bending stiffness can sometimes have beneficial effects [30]. A bending resonance of the soundpost gives a possible mechanism for such behavior. A poorly-fitted post will behave like a "pinned-pinned" beam, whose ends are able to rock where they touch the plates. A well-fitted post, under sufficient force, will behave more like a "clamped-clamped" beam, in which the ends can only rotate with the plates, not independently of them. This will change the frequency of the post resonance, but perhaps more importantly it allows this resonance to be efficiently coupled to the plate behavior and thus to produce audible consequences of some kind. Again, this seems a promising topic for investigation.

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DISCUSSION

It has been suggested in this article that many questions in violin acoustics can be illuminated by an awareness of the modal and statistical overlap factors. These give a simple, intuitive and measurable way to characterize the influence of damping and of variations between instruments. Different statistical overlap factors can be defined for different populations of instruments, relevant to different questions. A broad question like "What is a violin?" requires some knowledge of what features are in common between all respectable violins. A question like "How useful is plate tuning?" perhaps suggests measuring, and comparing, statistical overlap factors for two different sets of instruments: one set of careful geometric copies of an original model taking no account of wood variation, and a second set in which simple acoustical testing was used to guide systematic compensation via arching or graduation patterns for the variability of the wood.

The other general message of this article is that a description of individual vibration modes is useful for low frequencies, but that such a description ceases to be very illuminating above about 1 kHz or so. Instead, researchers should be looking for aspects of the behavior of a violin in the kilohertz range, which somehow transcend the complication of the many overlapping modes. These are the things that offer makers a chance to control the musical result, when they have no hope of controlling every individual mode. Some examples of such controllable features have been suggested, and no doubt others can be dreamed up. The most exciting challenge for researchers in the next decade might be to conceive and carry through projects, whether theoretical or experimental, which address questions of this type. ■ **CASJ**

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