Applied Acoustics **14** (1981) 455–469

AN INTRODUCTION TO STATISTICAL ENERGY ANALYSIS OF STRUCTURAL VIBRATION

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(Received 22 February, 1981)

SUMMARY

The ideas of the approach to vibration analysis called Statistical Energy Analysis (SEA) are explored without going into great technical detail. The aim of this description is to give guidance to those with particular vibration problems who may ask whether they should be using SEA and, if so, what expectations they should have of it. In the first section, SEA in its most common form is illustrated by the simplest example.

In the second section, the question of the underlying assumptions of SEA is considered by a simple and apparently novel approach. This discussion also gives some information on possible methods of measuring the SEA parameters in a given problem and deciding whether a SEA model is indeed appropriate for that problem. We also attempt to give guidance on how SEA should be applied to a given problem, especially how the system under study should be divided up into subsystems.

One area in which SEA can be especially useful is in the design of a systematic sequence of experiments on full or model scale when trying to pin down the source of a particular vibration problem. SEA can provide a succession of models, starting with the simplest possible and becoming progressively more complicated, which enable results to be interpreted and suitable questions to be asked for the next stage of testing. This is a more valuable service than is commonly appreciated: without such guidance, an experimenter faced with a complicated structure can spend a very long time making measurements which turn out not to address the problem in hand.

INTRODUCTION

Analysis of the vibratory behaviour of a complex structure can be undertaken in two basically different ways. If one is interested in gross movements of the structure on a

Applied Acoustics 0003-682X/81/0014-0455/$02.50

large scale—for example, to study the flexing of a ship's hull in a heavy sea—then detailed study of the lowest frequency vibration modes is appropriate A great deal is known about such analysis, largely as a result of the monumental work of Lord Rayleigh at the end of the last century [1] Indeed, theory has not advanced much since Rayleigh's day although the spectacular development of computing power in recent years has made brute-force numerical calculation of such modal behaviour in complex structures feasible for the first time (the finite element method, NASTRAN, etc)

If, on the other hand, the main interest is in the behaviour of the structure at higher frequencies, deterministic analysis of individual modes becomes less feasible and also less useful There are four reasons for this First, the modes crowd together in frequency so that many more of them need to be considered Secondly, higher frequency (i e shorter length-scale) modes are more sensitive to the inevitable small variations in structural detail even in nominally identical structures, so that they are harder to predict reliably Thirdly, related to the previous point, numerical accuracy decreases as one goes higher up the mode series so that, even if the real structure behaves like the model under study, the numerical predictions from the model may not be reliable Finally, even if an accurate and relevant numerical simulation can be made in the high frequency range under discussion there is the problem that computer output tends to be voluminous to the point of indigestibility when the model is complicated, so that it is hard to make use of the results In particular, for a model with many parameters, one needs some estimate of the sensitivity of the predictions to variations in all these parameters since they will be known only approximately for the real structure Usually the only way this is attempted in a numerical study is by changing each parameter in turn and running the program again, and it is clear that the more parameters there are, the more the problem of indigestibility of the results is multiplied

In the face of all these difficulties it is frequently more appropriate to use statistical techniques to average out in a suitable way the detailed modal behaviour one can then discuss such ideas as mean power flow between parts of the structure in a given frequency band or the expected spatial distribution of vibrational energy in the structure when broad band input is supplied at a localised place—for example, from an engine

The two approaches to the problem, deterministic and statistical, are not in competition for the great majority of applications statistical techniques take over from deterministic techniques, both in feasibility and usefulness, as the frequency range of interest rises through the mode series of the structure With the deterministic approach, as we have just said, computation of individual modal behaviour becomes increasingly difficult and unreliable as we go to higher mode numbers (beyond twenty or thirty, perhaps) The statistical approach, on the other hand does not require such detailed calculations and becomes increasingly successful as the resonance frequency spacing gets smaller (compared with the half-power bandwidth of each mode) the more modes one can average over, the more reliable the average becomes as an estimate of what actually happens in the structure There may be an intermediate frequency range where both approaches can be tried, but in such a situation it is possible that neither method will give entirely satisfactory results

The statistical method is paramount in architectural acoustics,[2] since the number of modes of a large auditorium within the audible frequency range run into tens of millions In such a situation one can expect statistics to be very reliable, while deterministic methods are obviously out of the question We cannot expect structural problems to present quite so many modes to work on, so, while the architectural acoustician can use statistical methods without a second thought, in structural problems we must take care in each situation to assess the expected reliability of our estimates Work is still in progress to enable this to be done more reliably than at present, but we do not discuss this here since our primary aim is to give a useful overview of the statistical approach, especially the particular technique known as SEA

THE STATISTICAL ENERGY METHOD AND ITS THERMAL ANALOGY

Having seen that for many practical problems deterministic analysis is not useful and statistical analysis must be used, we now discuss what kind of results a statistical analysis will give We discuss primarily the existing body of methods and results known as SEA,[3] but we should note that this represents only a particular class of possible techniques which are, in some sense, statistical Extensions and generalisations of conventional SEA to widen its scope of applicability are being developed in various places at present The recent review by Skudrzyk[4] is an example of a somewhat different statistical approach from the one we discuss here

The basic insight of conventional SEA is that, under most circumstances, energy in the form of vibration behaves in the same way as energy in the form of heat it diffuses from the 'hotter' places to the 'cooler' ones at a rate proportional to the difference of 'temperature', the constant of proportionality being a measure of 'thermal conductivity' Before explaining how this analogy is constructed, we recall the behaviour of thermal diffusion Consider a 'lumped' system consisting of two elements of sufficiently high conductivity that we may regard them as having uniform temperature, each of which can lose heat by radiation to the surrounding air and can also communicate heat to the other element via a connection of relatively low conductivity For simplicity we consider the two elements to be identical, one being supplied with heat from an external source, as illustrated in Fig 1 We are

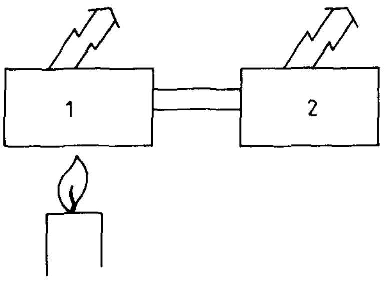

Fig 1 The simplest example of thermal diffusion, which illustrates the thermal analogue of SEA Two identical thermally conducting elements are connected by a link with lower conductivity Each element can also lose heat by radiation to the surroundings and one element is heated by an external source

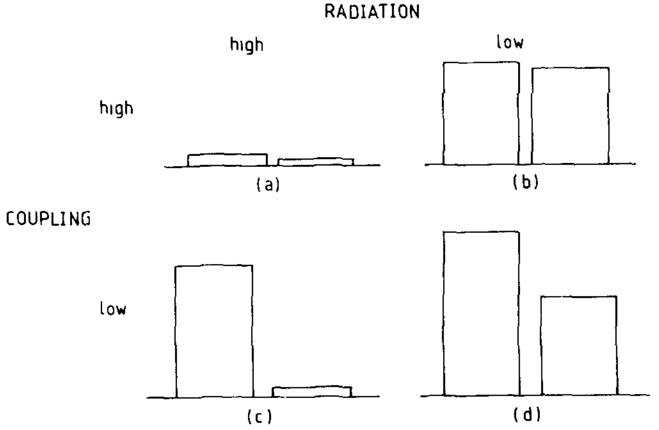

Fig 2 Temperatures of the two elements of Fig 1, under the four combinations of high and low radiative loss and high and low coupling conductivity for a given rate of heat input to element 1

interested in the equilibrium temperatures in the two elements which result, under the various conditions of high and low radiation loss and high and low coupling conductivity

These temperatures are illustrated in Fig 2 Notice that cases (a) and (d) give the same ratio of temperatures between the two elements, since this depends only on the ratio of radiative loss to coupling conductivity, which is the same in both cases The absolute magnitudes of temperatures are greater in case (d), of course, since, for a given rate of heat input, both elements must get hotter to compensate for the smaller value of radiative loss factor In case (b), the coupling is so strong compared with the radiative loss that the two elements are not really separate and both have approximately the same temperature (that is, we have approximate *equipartition* of energy between elements) In case (c), radiation is so much more effective than coupling between the elements that element 2 hardly knows that element 1 is being heated since the heat input is almost entirely balanced by radiation from element 1

To relate this thermal model to structural vibration, we consider a complex structure which consists of two coupled substructures These might, for example, be two plates separated by a stiffening beam, or the hull and one deck of a vessel Now, provided our substructures have an adequate number of modes within the frequency range in which we are interested, we can average over these modes and regard the *mean modal energy* in each substructure in that frequency range as a measure of the 'temperature' of the substructure We can then identify the parameters of the mechanical system with those of the thermal system as follows

(i) Thermal capacity of the element corresponds to *modal density*, that is, the number of modes whose frequencies fall within a given range
(ii) Radiative loss corresponds to damping of the vibration modes in that frequency range
(iii) Conductivity, or loss by coupling, corresponds to a measure of the strength of the mechanical coupling of the substructures (derived, for example, from relative impedance)

With these identifications, flow of vibrational energy in the structure behaves in the same easily visualised way as flow of heat in the thermal analogy

Thus we can now regard the four cases of Fig 2 as showing how the mean-square vibration level in the two substructures depends on the strengths of damping loss and coupling loss The source of heat in Fig 1 becomes a source of vibration, such as a machine attached to subsystem 1 We have, therefore, a very simple way of discussing the effect on vibration in subsystem 2 of changing the damping in either subsystem, or of isolating the two subsystems we can regard the attempt to minimise vibration level in subsystem 2 as the prototype for all noise and vibration control problems The description in vibration terms of the four cases of Fig 2 parallels the description given above in thermal terms for example, when coupling is strong compared with damping, approximate equipartition of energy amongst all the modes within each of the two subsystems results.

A general conclusion following from this is that vibration analysis by this statistical method will never give an answer which could not have been obtained, at least qualitatively, by *ad hoc* argument, in the same sense that thermal diffusion always follows one's intuition. The fact that any particular conclusion of SEA is likely to be, in qualitative terms, 'common sense' is not, however, a shortcoming of SEA. It is the main strength of SEA that it can be used easily to put such common sense arguments on a routine, quantitative footing. It must not be forgotten, after all, that a conclusion which is 'obvious' after it has been reached will not necessarily have seemed so obvious before that, in the absence of a standard method of approach which is guaranteed to find the obvious answer.

We can illustrate this by a particular case of the two-subsystem problem which we discussed above (Fig. 1). Suppose the two subsystems are quite strongly coupled together and that subsystem 1, which is directly driven, has much lower damping than subsystem 2, which is only driven indirectly. We can now ask whether, in order to reduce the vibration level in subsystem 2, we would be better advised to apply damping to the previously undamped subsystem 1 or to reduce the coupling by some isolation procedure. If we damp subsystem 1 somewhat not much benefit will accrue, since the total dissipation still has to balance the input and thus as subsystem 2 still has higher damping than subsystem 1, most of the energy will have to flow into that subsystem to be dissipated there. On the other hand, if we isolate the subsystems so that the coupling between them is weaker, again no benefit will accrue since, once again, the energy can only be dissipated in subsystem 2: in this case the vibration level in subsystem 1 will rise substantially without the level in subsystem 2 being changed. Thus, neither damping nor isolation does any good on its own. However, if we do *both together* we gain substantial benefit. In that case, the energy is confined largely in subsystem 1 because of the isolation, and this confinement makes the damping on that subsystem much more effective than it was previously, and now a substantial proportion of the energy being injected will be dissipated in subsystem 1 without reaching subsystem 2. Examples of similar behaviour have been discussed by Maidanik.[5]

The fact that SEA always produces answers which should not be surprising is another way in which it differs from large-scale numerical modelling of structures. If a finite element program gives a result which suggests that there will be a 'hot spot' in some region of the structure, one cannot be sure that this is a real effect and not an artifact of the numerical method. SEA, on the other hand, cannot give predictions in which the mean modal energy increases in some region without a direct cause, so that if it predicts a hot spot it should be right.

We have described in some detail SEA and its thermal analogy in the context of the simplest problem, with just two subsystems. The method is, of course, applicable to problems with many subsystems, and much the same kind of conclusions will be reached in that general case We will consider this in more detail in the next section, where some discussion of the range of applicability of SEA will be given, together with one possible approach to the measurement of the SEA parameters for a given structure Before doing that, however, we note one particular area in which SEA can be of use, in designing and interpreting a series of experiments in an effort to understand and ameliorate a vibration problem on a particular structure

If an experimenter is given such a task he needs some guidance on how to approach the problem, since there will inevitably be an impossibly large number of measurements which could be taken and which bear on the problem in some way Provided the terms of reference of SEA are appropriate to the problem in question (i e provided an answer in terms of mean-square vibration amplitudes over various subsystems would be useful), SEA can be used to provide specific, simple questions for the experimenter to tackle One simply asks, 'What is the simplest possible division of our structure into subsystems which holds out any possibility of modelling the problem in hand?' In many problems, the answer may be that just two subsystems are appropriate in the first instance, although more may be needed in other problems

The effort to fit a SEA model with that number of subsystems to the observed behaviour of the whole structure gives usefully simple goals for the first round of measurements Possibly this alone will solve the problem, but it is perhaps more likely that the measurements cannot be made to agree with the simple model This does not mean that SEA is wrong, or that effort has been wasted On the contrary, only by asking the specific questions prompted by the SEA model has this disagreement been revealed, and the disagreement itself probably conveys useful information about the problem since it shows that the first guess at what was needed to model the problem was inadequate One can then ask what is the simplest modification to the model which might account for the discrepancy and repeat the cycle with the next most complicated SEA model

This progression of models, starting from the simplest possible, can provide the necessary framework for the organisation of ideas and the interpretation of results to enable the problem to be solved in the shortest time This last point is worth stressing much of the emphasis in the literature of SEA, particularly in the book and many papers of Lyon,[3] has been on the use of SEA specifically as a predictive tool In the use of SEA described above, however, while prediction plays a part, perhaps the greatest benefit of using SEA is the language and mental framework it provides for taking an apparently complicated problem and finding simple questions to ask about it SEA does not give a recipe for getting accurate estimates of vibration levels in all situations with no effort, nor does any other possible approach Problems are solved by the interaction of the predictions of deliberately simplified models with measurements on the full-scale structure or on models

DETERMINATION OF SEA PARAMETERS BY THE INVERSE PROBLEM, AND TESTING THE VALIDITY OF A SEA MODEL

Having described the terms of reference of a SEA model and given some impression of the sort of answer which it can be expected to give, we now examine the circumstances under which we expect a SEA model to be valid We are also interested in how SEA parameters might be determined In the existing SEA literature,[3] including the previous work of the present author,[6] the question of the applicability of SEA has generally been approached by first considering the power flow between individual modes of subsystems, then making appropriate statistical approximations to obtain statements about the average power flow between the subsystems In doing such calculations, a set of conditions sufficient to guarantee the validity of the SEA model is found, and these conditions are frequently very restrictive Thus the impression is given that SEA is only valid under very limited conditions In what follows we give an account of this problem from a different point of view We think that this is both useful in itself and provides a counterweight to the other method since it shows that SEA is, in fact, valid under rather general circumstances

Suppose that we have a system composed of a number, N, of coupled subsystems as suggested in Fig 3 We examine the form of a general SEA model of this system

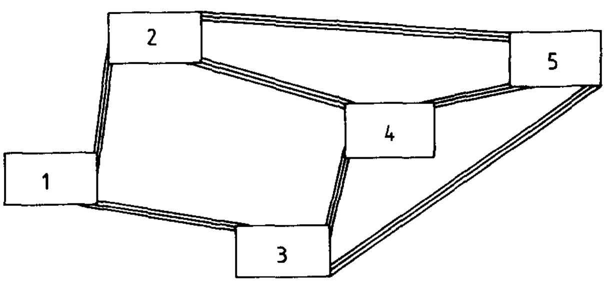

Fig 3 Schematic representation of coupled subsystems making up the complete system whose vibrational behaviour is under study

We write E_i for the average energy per mode in subsystem i, and P_i for the rate of energy input to that subsystem from external sources (We recall that average energy per mode is the correct quantity, since we want the analogue of temperature rather than of thermal capacity)

Three assumptions are made in standard SEA modelling The first is that the rate of energy dissipation by subsystem i is proportional to the energy, E_i We shall call the proportionality constant S_{ii} (clearly, this constant is positive because energy is being dissipated, not created The second assumption is that the rate of power flow from subsystem i to subsystem j is proportional to the difference of their energies we take it to equal $S_{ij}(E_i - E_j)$ Again, it is clear that the constants S_{ij} are all non-negative (some may well be zero, of course, since some subsystems will not be directly coupled to others at all) The third assumption is that the driving forces on the different subsystems are statistically independent so that we can add the energy responses of a given subsystem produced by these different driving forces to obtain the total mean modal energy of that subsystem

Energy balance on subsystem i now requires that

$$S_{ii}E_i + \sum_{j \neq i} S_{ij}(E_i - E_j) = P_i \tag{1}$$

where the sum over j represents all energy flow away from subsystem i to other subsystems (Those who are familiar with electrical circuit theory will now be aware of an alternative analogue of SEA to the thermal one discussed above eqn (1) is a statement of Kirchhoff's law)

Our eventual aim is to use the SEA model to predict the change in vibrational behaviour of the whole system when some modification is made to the structure To do this we need to know the energies, E_i, in response to given rates of external energy input, P_i At present, eqn (1) expresses the P_i's in terms of the E_i's, so we must invert this relation To achieve this, we first rearrange eqn (1) by collecting all occurrences of each E_i together, to read

$$P_i = \sum_{j=1}^{N} X_{ij}E_j \tag{2}$$

where

$$X_{ij} = \begin{cases} -S_{ij} & i \neq j \\ \displaystyle\sum_{k=1}^{N} S_{ik} & i = j \end{cases} \tag{3}$$

We now invert the matrix of eqn (2) to obtain the desired form of predictions

$$E_i = \sum_{j=1}^{N} A_{ij}P_j \tag{4}$$

We have written A for the inverse of X we shall discuss below the physical interpretation of matrix A

From this form of the SEA model of the system, we can note an important general point Since both matrices A and X are symmetric, the model expressed by matrix X has precisely the same number of parameters as the number of predictions sought (although we should note that, in practice, with many subsystems, we expect some pairs of subsystems to have no direct coupling so that certain off-diagonal elements of X will be zero), expressed in matrix A This immediately suggests that the conditions of applicability of the SEA model cannot be very restrictive, since we have so many adjustable parameters in our model Since the model is not substantially more compact than the predictions it yields, we must ask what benefit we derive from the SEA model why can we not proceed straight to the matrix A?

The answer to this lies in the fact that we have a more direct and useful physical interpretation of the terms of X, since we can readily derive from them the terms of matrix S, of which the diagonal elements express the damping in the individual subsystems, while the off-diagonal elements describe the strength of coupling between pairs of subsystems Thus, if we are using the SEA model to study how the vibrational behaviour of the whole system will change when some modification is made to the structure, we would expect to be able to describe that structural change in terms of one, or at most a few, elements of X changing Having characterised the change in X, we can then, of course, use the model to evaluate the change in A If we were trying to predict A directly, however, we would have to contend with the fact that any change in the structure will in general change *all* the elements of A Thus the SEA model can give useful insight into the sensitivity of the vibrational behaviour of the system to such structural modifications

Having seen that the SEA model is worth having, we need to answer two questions—When is a SEA model valid? How do we evaluate the model parameters S_{ij}? We approach these questions simultaneously by asking how one could determine *experimentally* whether an SEA model is valid We imagine performing a set of measurements on this system, in which we drive one subsystem at a time, and measure the response of all the subsystems in each case Thus, if we drive subsystem i with a rate of energy input scaled to unity, we can determine the mean energy per mode in subsystem j, which will be the matrix element A_{ij} Thus, the matrix A is directly observable If, now, all the subsystems are driven simultaneously with the energy input rates, P_i, provided we make the single assumption that the driving forces on the different subsystems are statistically independent, we can superimpose these energy responses to obtain the total mean modal energy of subsystem i in the form of eqn (4)

Thus we expect the subsystem responses to depend on the inputs to the various subsystems in a simple matrix fashion under very general circumstances the question of the validity of the SEA model hinges on whether this matrix has the appropriate form of inverse to look like a SEA model as in eqn (3) The only constraints on the applicability of SEA to the system in question lie in the *signs* of the matrix entries A is composed entirely of non-negative terms, while X has all its off-diagonal terms non-positive and all its diagonal terms positive and sufficiently large that the sum along any row is non-negative A SEA model is thus valid for any system whose matrix A has an inverse in the form X

To evaluate precisely the conditions on the matrix A which guarantee such an inverse would take us into research problems in linear algebra, beyond the scope of this introductory account We thus content ourselves here with two simple examples of these conditions, for the case of just two subsystems and for the case of very weakly coupled subsystems For the two-subsystem case, we suppose our matrix A has the form

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{5}$$

where a, b and c are all positive numbers Then the inverse is

$$\frac{1}{\Delta}\begin{pmatrix} c & -b \\ -b & a \end{pmatrix} \tag{6}$$

where

$$\Delta = ac - b^2$$

Hence any such matrix A has an inverse whose diagonal terms are positive and whose off-diagonal terms are negative provided that $\Delta > 0$ Our other condition is that the sum of any row of the inverse matrix should be positive, and this requires

$$a > b \qquad c > b \tag{7}$$

which is also sufficient to ensure the condition $\Delta > 0$ Thus we find that for a system composed of just two subsystems, the only condition necessary for the validity of an SEA model in the terms formulated here is that in the observed matrix A, whenever one subsystem only is driven, that subsystem should respond with greater mean modal energy than the other, non-driven, subsystem This condition is physically very plausible and we can deduce that SEA should always be applicable to any problem with just two subsystems

For the case of weakly coupled subsystems, the matrix A will have the diagonal element much larger than all the off-diagonal elements in each row, since the driven subsystem will always vibrate much more vigorously than the others We shall show that if this diagonal dominance is sufficiently strong, the inverse matrix will necessarily be in SEA form Write the matrix A as the sum of a diagonal matrix D and a matrix E whose diagonal elements are zero and all of whose off-diagonal elements are much smaller than a typical diagonal element of D Then first-order perturbation theory says that the inverse of A is given approximately by

$$A^{-1} \simeq D^{-1} - D^{-1}ED^{-1} \tag{8}$$

which has the correct sign pattern for a SEA model and also satisfies the constraint of the sum of any row being non-negative provided the diagonal dominance of A was sufficiently strong (i e the coupling sufficiently weak) Thus sufficiently weak coupling, like the case of two subsystems, guarantees the correct form of inverse matrix and thus the applicability of a SEA model without further conditions

With many subsystems and stronger coupling, things are more complicated and we do not discuss the problem in detail Returning to our imagined experiment on such a system, we can now enquire a little more closely how we should test our measured matrix A to determine whether a SEA model would be valid and, if so, how to determine the SEA parameters We first evaluate A^{-1} to find out whether it is in the form X If it is, all is well If it is not, however, we do not necessarily give up and conclude that SEA is not usable There will be errors in the measurements of A_{ij}, so we must next ask whether we can find a modification of A within the error bands of the measurements which does have an inverse in the correct form Once we have found such an A, then all the SEA parameters are determined since, from X, we can deduce S, which is composed of the damping loss factors and coupling loss factors called for in the SEA model

We illustrate this procedure of searching for a matrix close to the measured A, but which has an inverse in the correct form, by a simple numerical example Suppose we have three subsystems and our measured matrix A had the values

$$\begin{pmatrix} 1\,0 & 0\,6 & 0\,9 \\ 0\,6 & 1\,0 & 0\,5 \\ 0\,9 & 0\,5 & 1\,0 \end{pmatrix} \tag{9}$$

The inverse of this is

$$\begin{pmatrix} 6\,25 & -1\,25 & -5\,00 \\ -1\,25 & 1\,58 & 0\,33 \\ -5\,00 & 0\,33 & 5\,33 \end{pmatrix} \tag{10}$$

The only term which prevents this being in the correct form is the (2, 3) entry 0 33, which is positive If we simply set this entry to zero as a first guess at an SEA matrix close to the actual inverse, and invert that matrix again to compare with the original matrix (9), we obtain

$$\begin{pmatrix} 1\,74 & 1\,38 & 1\,63 \\ 1\,38 & 1\,72 & 1\,29 \\ 1\,63 & 1\,29 & 1\,72 \end{pmatrix} \tag{11}$$

which is a long way from matrix (9) and is therefore not acceptable If, however, we now use a simple hill-climbing computer program to find the matrix X_{ij} in the SEA form (3) whose inverse best fits matrix (9) in the sense of minimising the sum of squares

$$\sum_{ij} (A_{ij} - X_{ij}^{-1})^2 \tag{12}$$

we find that the matrix

$$\begin{pmatrix} 6\ 28 & -1\ 03 & -4\ 97 \\ -1\ 03 & 1\ 71 & 0\ 0 \\ -4\ 97 & 0\ 0 & 5\ 34 \end{pmatrix} \tag{13}$$

has an inverse

$$\begin{pmatrix} 0\ 97 & 0\ 60 & 0\ 89 \\ 0\ 60 & 0\ 96 & 0\ 56 \\ 0\ 89 & 0\ 56 & 1\ 01 \end{pmatrix} \tag{14}$$

which is a great deal closer to the original matrix A of eqn (9) than was our crude first guess, although matrix (13) is very close to matrix (10) Thus we have found a SEA type matrix which produces predictions very close to our supposed observations and quite possibly within the permitted measurement errors of those observations Thus we feel that a SEA model with these parameters is probably appropriate for investigating the effect on the vibrational behaviour of our system of any changes of the structure, which would change individual entries of the matrix X in a way which we might hope to be able to predict

This example illustrates clearly that small changes in a matrix can produce very large changes in the inverse matrix This observation is relevant to us in several ways In the problem we have been discussing of determining whether a SEA model fits a given set of measurements it warns us against hasty judgements But it is also relevant to our general problem of using SEA to analyse the effect on the vibrational behaviour of a structure of changes in the structure the changes in response (A) can be surprisingly great for a small change in subsystem coupling or damping (X)

We should now ask to what extent our imagined experiment, described above for the purposes of understanding SEA, can form the basis of real experiments when using SEA It seems to the present author that this could be a very sensible thing to do in certain circumstances, although it is very different from what is usually advised in the standard SEA literature There are two main points to be made

First, it all depends on our purpose in wanting to use SEA If we are designing an entirely novel structure and are wanting to produce vibration estimates *a priori* from the drawing board, then an experimental procedure is clearly not appropriate Much of the SEA literature seems to be slanted towards such uses However, how often is this done? Usually new structures differ only in details from existing ones of the previous generation, or at least from models if there is no previous generation We then have available the existing structure to make measurements on, and it is surely sensible to find out whether SEA is indeed applicable to that existing structure with our chosen decomposition into subsystems before asking whether we can use SEA to study changes in the structure? The method described above can do this reasonably easily Having done that, and in the process determined all the SEA model parameters for the existing structure, we are in a position to investigate the changes in those parameters resulting from our proposed structural changes, and thus to study the effect on vibrational response

This brings us to the second main point to be made much of the emphasis in the SEA literature is on *calculating* coupling loss factors—that is, elements of S_{ij}—whereas here we are advocating measuring them all Does this not represent an unnecessarily large experimental effort? The answer is, if one studies the SEA literature closely, that for realistic structure–structure coupling problems, coupling loss factors are very hard to predict reliably on theoretical grounds, and one has to resort to measurement in any case (see especially the chapter in Lyon's book[3] on determining coupling loss factors) If one is going to have to resort to measurement in any case, the author would suggest that the procedure described above could be simpler and more systematic, as well as more informative, than the approaches suggested by Lyon and others

Having made the point that coupling loss factors have to be measured rather than calculated, we must now address the question of whether it is therefore impossible to predict the effect on the elements of X of changes in the structure Fortunately, this need not be the case While it may be very hard to predict these values *a priori* with no experimental information, it can be very much easier to predict, at least to an acceptable approximation, the dependence of a given term on a particular parameter of the structure such as a plate thickness usually some power-law dependence can be deduced from simple modelling of the structure Thus, with the experimental determination of the coupling loss factors on the unmodified structure as a calibration, we can hope to predict the new values of these factors after a modification of plate thickness, or whatever, without too much difficulty

To sum up the discussion of this section, we have argued that SEA is likely to yield a valid model for a large range of structures, at least as a reasonable approximation We have given an experimental procedure for testing whether this is indeed the case on the particular structure in question, and this experimental procedure also yields values of the SEA model parameters for the structure The procedure consists of driving one subsystem at a time with an appropriate source and measuring the energy responses of all the subsystems to that driving The matrix of measurements thus obtained is then tested for compatibility with an SEA model by finding the matrix in SEA form (eqn (8)) whose inverse best approximates the measured matrix The closeness of fit achieved in this search measures the credibility of the SEA model In this fitting process we can allow for any additional knowledge we may have of the system—for example, by forcing terms to zero which correspond to coupling between subsystems which have no direct connection or by constraining certain terms to remain equal if we have identical subsystems identically coupled If we find that a SEA model fits well and have thus determined all the model parameters, we then have a good chance of being able to predict the effect on those model parameters of changes which might be made to the structure The SEA model should then give a reasonable estimate of the effect of those structural changes on the vibrational response of the structure

ACKNOWLEDGEMENT

This paper has benefitted greatly from discussions with many colleagues, especially D Baldwin, C H Hodges and I Roebuck The work was supported by the Procurement Executive, Ministry of Defence

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